

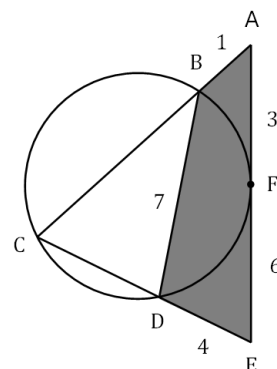
Theta

1. Let $X = 0.\overline{126} \rightarrow 10X = 1.\overline{26}$ & $1000X = 126.\overline{26} \rightarrow 990X = 125 \rightarrow X = \frac{125}{990} = \frac{25}{198}$. So $A + B = \boxed{223}$

2. $2 + 3 + 5 + 7 = \boxed{17}$ and you don't have to try after that.

3. $x = \sqrt{6 + 11\sqrt{6 + 11\sqrt{6 + 11\sqrt{\dots}}}} \rightarrow x^2 = 6 + 11x$, which has roots $\frac{11 \pm \sqrt{145}}{2}$. Finally the original expression is clearly positive so we choose only the root $\boxed{\frac{11 + \sqrt{145}}{2}}$.

4. We will use the fact that the area of $ABDE$ is the area of $\triangle ACE$ minus the area of $\triangle BCD$. Using Power of a Point on $\overline{AF}$ and $\overline{AC}$, we have $3^2 = (1)(1 + BC)$, so $BC = 8$. Similarly with $\overline{EF}$ and $\overline{EC}$, we have $6^2 = (4)(4 + DC)$, so $DC = 5$. Now, we note that $\triangle ACE$ is equilateral with side length 9, so $\angle ACE = 60^\circ$. We use the formula for area of a triangle given two sides and an angle between them: $\text{Area} = \frac{1}{2}ab \sin C$. The area of $\triangle ACE$ is simply $\frac{9^2\sqrt{3}}{4} = \frac{81\sqrt{3}}{4}$, and the area of $\triangle BCD$ is $\frac{1}{2}(5)(8)(\sin 60^\circ) = 20\left(\frac{\sqrt{3}}{2}\right) = 10\sqrt{3}$. Subtracting these gives the final area of $\frac{81\sqrt{3}}{4} - \frac{40\sqrt{3}}{4} = \frac{41\sqrt{3}}{4}$.



The x, y, z values fit all the conditions given, so $x + y + z = 41 + 3 + 4 = \boxed{48}$.

5. The remainder of a polynomial divided by $x - a$ is $P(a)$. With this fact in mind, we proceed: $P(1) = 5$ and $P(-1) = 5$

It is also given that $P(0) = 3$. Since $P(x)$ is monic, $P(x) = x^3 + ax^2 + bx + c$ for unknown constants a, b , and c .

$$P(0) = 3 \rightarrow 0^3 + a \cdot 0^2 + b(0) + c = 3 \rightarrow c = 3$$

$$P(1) = 5 \rightarrow 1 + a + b + c = 5 \rightarrow a + b = 1$$

$$P(-1) = 7 \rightarrow -1 + a - b + c = 7 \rightarrow a - b = 5$$

Solving for a, b , and c gives $a = 3, b = -2$, and $c = 3$, so $P(x) = x^3 + 3x^2 - 2x + 3$.

We wish to find $P(3) = (3^3 + 3 \cdot 3^2 - 2 \cdot 3 + 3) = \boxed{51}$.

Alpha

6. Amplitude = $\sqrt{(\sqrt{26})^2 + (\sqrt{10})^2} = \boxed{6}$.

7. These points all lie on a circle of radius $4^{\frac{1}{2015}} \approx 1$, and the polygon almost coincides completely with this circle because there are so many points. So the area will be very close to that of the unit circle, π . And $\lfloor 10\pi \rfloor = \boxed{31}$

8. By Cramer's Rule, $w = \frac{\begin{vmatrix} 3 & 2 & -1 & 4 & -5 & 6 \\ 1 & 3 & 0 & 5 & -4 & 2 \\ 2 & 4 & -2 & 0 & 3 & 6 \\ 0 & 5 & 1 & -1 & -2 & 2 \\ 5 & 6 & 3 & 7 & 0 & 6 \\ 1 & 7 & 5 & 1 & -1 & 2 \end{vmatrix}}{\begin{vmatrix} 3 & 2 & -1 & 2*4 & -5 & 6 \\ 1 & 3 & 0 & 2*5 & -4 & 2 \\ 2 & 4 & -2 & 2*0 & 3 & 6 \\ 0 & 5 & 1 & 2*(-1) & -2 & 2 \\ 5 & 6 & 3 & 2*7 & 0 & 6 \\ 1 & 7 & 5 & 2*1 & -1 & 2 \end{vmatrix}} = \frac{1}{2}$. The coefficients of w match twice the right hand side of the equal sign every time, we know by determinant rules $w = \boxed{\frac{1}{2}}$.

9. $\tan(\arctan(\alpha) + X) = \frac{\tan(\arctan(\alpha)) + \tan(X)}{1 - \tan(\arctan(\alpha))\tan(X)} = \frac{\alpha + \tan(X)}{1 - \alpha \tan(X)} = \frac{\alpha + \frac{\tan(\arctan(\beta)) + \tan(\arctan(\gamma))}{1 - \tan(\arctan(\beta))\tan(\arctan(\gamma))}}{1 - \alpha \frac{\tan(\arctan(\beta)) + \tan(\arctan(\gamma))}{1 - \tan(\arctan(\beta))\tan(\arctan(\gamma))}} =$
 $\frac{\alpha + \frac{\beta + \gamma}{1 - \beta\gamma}}{1 - \alpha \frac{\beta + \gamma}{1 - \beta\gamma}} = \frac{\alpha - \alpha\beta\gamma + \beta + \gamma}{1 - \beta\gamma - \alpha\beta - \alpha\gamma} = \frac{(\alpha + \beta + \gamma) - (\alpha\beta\gamma)}{1 - (\beta\gamma + \alpha\beta + \alpha\gamma)} = \frac{20 - (1)}{1 - 26} = \boxed{-\frac{19}{25}}$

10. The eccentricity is the ratio of the distance from the vertex to the focus to the distance from the vertex to the directrix. Therefore the vertex will be $\frac{2}{5}$ the distance from the focus to the directrix. The equation of the line coincident with the major axis is $y = \frac{1}{2}x$ since it must go through the focus and be perpendicular to the directrix. This line intersects the directrix when $-2x + 3 = \frac{1}{2}x \rightarrow x = \frac{6}{5} \rightarrow y = \frac{3}{5}$. The point $\frac{2}{5}$ the distance from the origin to $(\frac{6}{5}, \frac{3}{5})$ is $(\frac{2}{5} \cdot \frac{6}{5}, \frac{2}{5} \cdot \frac{3}{5}) = (\frac{12}{25}, \frac{6}{25})$. $\frac{12}{25} + \frac{6}{25} = \boxed{\frac{18}{25}}$

Mu

11. $\sqrt[3]{26} \approx \sqrt[3]{27} + f'(27) \cdot (26 - 27) = 3 - f'(27)$ where $f(x) = \sqrt[3]{x} \rightarrow f'(27) = \frac{1}{3\sqrt[3]{27^2}} = \frac{1}{27}$.
 So $\sqrt[3]{26} \approx 3 - \frac{1}{27} = \boxed{\frac{80}{27}}$.

12. The equations for the optimization are $2w + h = X$ and therefore $A = wh = w(X - 2w) = Xw - 2w^2$. So $A' = X - 4w = 0 \rightarrow w = \frac{X}{4} \rightarrow A = \frac{X^2}{4} - \frac{X^2}{8} = \frac{X^2}{8} = 200 \rightarrow X^2 = 1600 \rightarrow X = \boxed{40}$

13. Let $u = \sqrt{1 - \sqrt{1 - x}} \rightarrow x = 1 - (1 - u^2)^2 = 2u^2 - u^4 \rightarrow dx = (4u - 4u^3)du$. Also $x = 0 \rightarrow u = 0$ and $x = 1 \rightarrow u = 1$. So $\int_0^1 \sqrt{1 - \sqrt{1 - x}} dx = \int_0^1 (4u^2 - 4u^4) du = \left[\frac{4}{3}u^3 - \frac{4}{5}u^5 \right]_0^1 = \boxed{\frac{8}{15}}$.

14. We are looking for points $(x_1, x_1^2 + 1)$ and $(x_2, -x_2^2)$ such that $2x_1 = -2x_2 = \frac{x_1^2 + 1 + x_2^2}{x_1 - x_2}$. The first equality implies $x_1 = -x_2$ so plugging that into the final term yields $2x_1 = \frac{2x_1^2 + 1}{2x_1} \rightarrow 2x_1^2 = 1 \rightarrow x_1 = \frac{\sqrt{2}}{2}$. Therefore the slope is $2x_1 = \sqrt{2}$ and one of the points is $\left(\frac{\sqrt{2}}{2}, \frac{3}{2}\right)$. So the equation of the tangent line is $y - \frac{3}{2} = \sqrt{2}\left(x - \frac{\sqrt{2}}{2}\right) \rightarrow \boxed{y = \sqrt{2}x + \frac{1}{2}}$.

15. Let $u = \frac{x}{2026} \rightarrow 2026 du = dx \rightarrow \int_0^{2026} \frac{x - 2026}{\ln\left(\frac{x}{2026}\right)} dx = 2026^2 \int_0^1 \frac{u - 1}{\ln(u)} du$. Consider $I(t) = \int_0^1 \frac{u^{t-1}}{\ln(u)} du \rightarrow I'(t) = \int_0^1 \frac{\ln(u) u^t}{\ln(u)} du = \int_0^1 u^t du = \frac{1}{t+1} \rightarrow I(t) = \ln(t+1) + C$. Clearly when $t = 0, I(t) = 0 \rightarrow C = 1$ so $I(t) = \ln(t+1) \rightarrow \int_0^1 \frac{u-1}{\ln(u)} du = I(1) = \ln(2)$. So the answer is $2026^2 \ln(2) = \boxed{4104676 \ln(2)}$

Open

16. The probability is $1 - \left(\frac{365}{365}\right)\left(\frac{364}{365}\right)\left(\frac{363}{365}\right) \dots \left(\frac{365-n+1}{365}\right)$. This is known to exceed 50% at $n = \boxed{23}$.

17. There are 23 problems on Hilbert's List, and the list of their statuses is available here: https://en.wikipedia.org/wiki/Hilbert%27s_problems. So $23 + 10 = \boxed{33}$.

18. This quote is from the LEGO Movie, and the numbers missing are 8 and 14, for a total of $\boxed{22}$.

19. The width of the Uniform distribution is $\gamma + \frac{6}{\sqrt{n}} - \gamma + \frac{6}{\sqrt{n}} = \frac{12}{\sqrt{n}} = \frac{12}{\sqrt{9}} = 4$, and the average value is clearly γ , so if the middle 95% of the distribution is from $\gamma - a$ to $\gamma + a$ then $0.95 = (2a)\left(\frac{1}{4}\right) = \frac{a}{2} \rightarrow a = 1.90$. Therefore with confidence 95%, $\gamma - 1.90 < g < \gamma + 1.90 \rightarrow \gamma - 1.90 < 10 < \gamma + 1.90 \rightarrow -11.90 < -\gamma < -8.10 \rightarrow 8.10 < \gamma < 11.90$. So the answer is $\boxed{(8.1, 11.9)}$

20. Clearly, the divisibility rules for numbers through 11 will not yield a positive result.

For 13, after checking 331 (prime), alternating triples will look like one of: $331-3=328$ or $331-33=298$ or $331-333=-2$ or $331-333+3=1$ or $331-333+33=31$ or $331-333+333=331$. The pattern repeats after that. None are divisible by 13 so no number of this form can be.

For 17, we multiply the last digit by 5 and subtract it from the remaining digits sequentially: $3\dots3 - 5 = 3\dots328$; $3\dots32-40=3\dots3288$; $3\dots328-40=3\dots3288$ etc. So 17 may be possible. Since the last digit is 1, and $17*3=51$, if we can get a five in long division we are done. Doing long division on a string of threes of arbitrary length, we get 1960784 after which we carry a 5 and can drop down a 1 for 51. So the number is $19607843*17=\boxed{333,333,331}$.