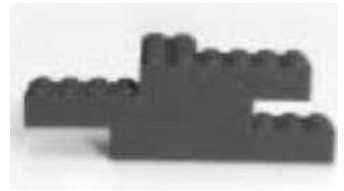
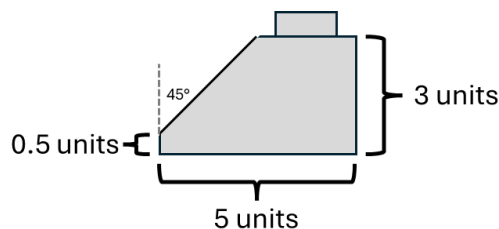
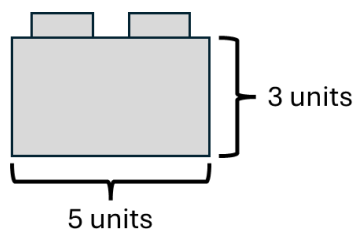


I. LEGO Math & Bash

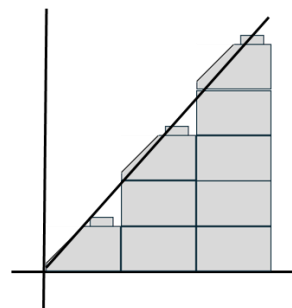
1. A bucket contains 4 yellow LEGO bricks, 3 red LEGO bricks, and 3 blue LEGO bricks. If you take all the bricks out of the bucket one at a time, what is the probability that the fourth and fifth brick you take out will be a yellow brick and red brick in that order? (2 points)
2. A LEGO knight has 4 different weapons, 3 different shields, and 5 different capes (which are all considered accessories for this problem). The knight can use at most one of each type of accessory (i.e. one weapon, one shield, and one cape), but he wears at least 1 accessory. How many assortments of accessories can the knight wear given these constraints? (2 points)
3. I have two identical 2x4 LEGO bricks. I wish to stack one on the other, so they are connected by at least one stud. If reflections of the resulting stack are distinguishable, but rotations are not, how many possible ways can this be done? (3 points)
4. Now I have six identical 2x4 LEGO bricks. I wish to stack one on top of the other, so that each is connected by at least one stud, and the total stack is six bricks high. If reflections of the resulting stack are distinguishable, but rotations are not, how many possible ways can this be done? (5 points)
5. The LEGO Group reported the answer to the previous question (offset by four due to calculator rounding) as “the number of ways to combine six two-by-four LEGO bricks of the same color.” In fact, this is only for stacks of height exactly six. When considering shorter stacks such as the image to the right. In 2005, a professor at the University of Copenhagen named Soren Eilers calculated the correct number of ways to do this for all possible heights two through six (note that height one isn't possible because the bricks must be connected). He needed to use a computer to do this, however. You can accomplish it by concatenating the answers to the following three math problems in either forward or backwards order (3 points):


- a. (3 points) The answer to this question is $[A + 1]$, where A is the area enclosed by the graph of
$$5x^2 - 8xy - 8\sqrt{2}x + 5y^2 + 10\sqrt{2}y = 719$$
- b. (2 points) Liz wishes to construct a 95% confidence interval for a population proportion with a margin of error of 0.0966 when using a critical value rounded to two decimal points. What is the largest possible sample size she will need to accomplish this?
- c. (4 points) If $\int_0^{\frac{\pi}{2}} \cos^n(x) \sin^5(x) dx = \frac{1}{96702120}$, find n .
6. Five LEGO minifigures with distinct heads, legs, and torsos are broken up into their heads, legs, and torsos. Heads are put into one bag, torsos a second, and legs a third. A minifigure are recreated by choosing uniformly at random a head, torso, and legs. What is the probability that the head, torso, and legs of the new minifigure were all part of different minifigures originally? (2 points)

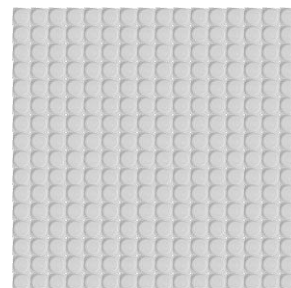
7. Referring to the previous question, after the first new minifigure is made, a second is made using the remaining parts, then a third, then a fourth, and then a fifth. Among the five new minifigures, what is the probability that none of them have two parts that were originally in the same minifigure? (6 points)
8. Gaby has two kinds of LEGO bricks available to her: 1x2 bricks and 1x2 slope bricks as shown below from the side with correct dimensions:



She wishes to approximate the area between certain curves and the x -axis by stacking these bricks on the x -axis and measuring their area (ignoring the studs on top). For example, to the right is one (not necessarily the best) approximation of $y = x + \frac{1}{2}$ from $x = 0$ to $x = 15$. Find the closest approximation possible using this approach in each of the following areas (3 points each):



- a. $y = 2500 - x^2$ from $x = 0$ to $x = 50$
- b. $y = 10 \sin\left(\frac{\pi x}{10}\right)$ from $x = 0$ to $x = 10$
9. The geometry of LEGO plates yields some surprising results (3 points each):
- a. A 16x16 LEGO plate (right) can be rotated clockwise so that exactly two studs on the rotated plate align with studs in the first row of the unrotated plate. When this is done, how many total studs align between the two plates?
- b. Referring to part a. above, find the tangent of the smaller of the two obtuse angles formed between the edges of the plates.



10. Let $\mathcal{L}$ be the set of all possible LEGO builds, let $\ell \in \mathcal{L}$, let $\#\ell$ be the number of individual LEGO pieces in ℓ , and let $F(\ell) = \lfloor \log_{10}(\#\ell) \rfloor + 1$. For example,

$$F(\text{LEGO brick}) = 1$$

$$F(\text{LEGO Star Wars set}) = 3$$

Evaluate each of the following (1 point each):

a. $F(\text{LEGO Eiffel Tower})$

b. $F(\text{LEGO World Map})$

c. $F(\text{LEGO Space Station})$

d. $F(\text{LEGO City set})$

e. $F(\text{LEGO Technic set})$

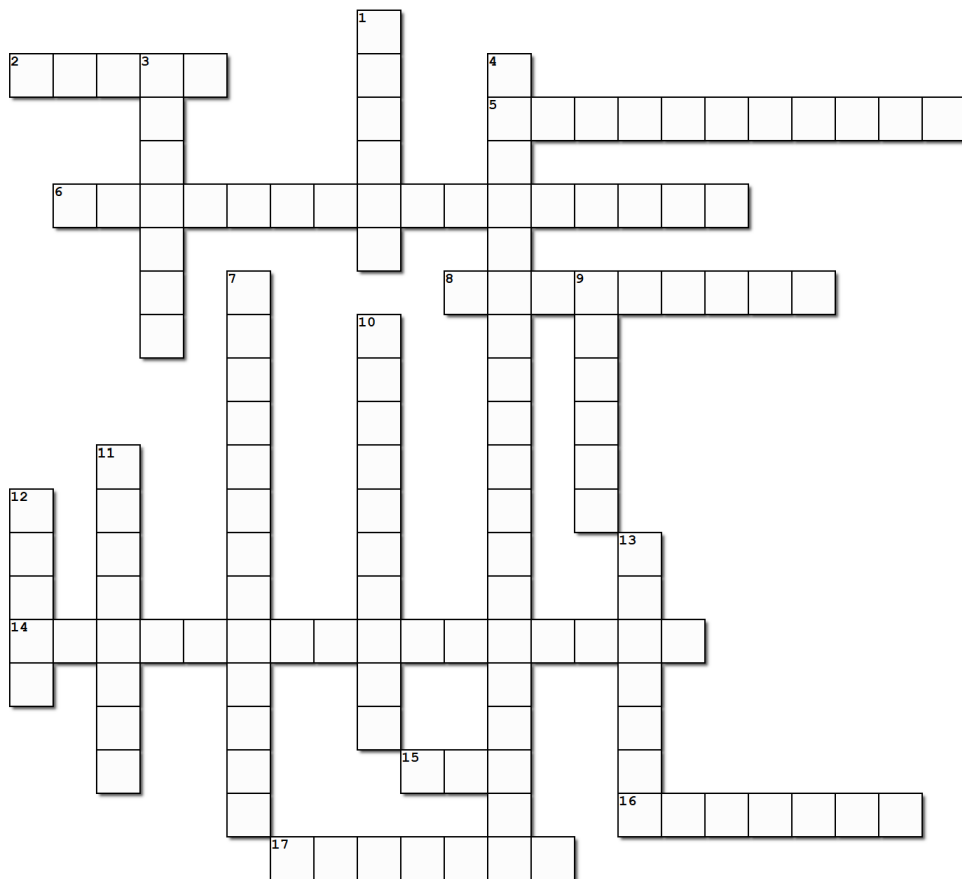
f. $F(\text{LEGO Einstein head})$

g. $F(\text{LEGO Technic robot})$

h. $F(\text{LEGO Tree set})$

II. LEGO x Words (2 points each)

Use the following crossword puzzle to solve the questions below. Also put the answers on your answer sheet!



Created using the Crossword Maker on TheTeachersCorner.net

Across

2. The product line of LEGO intended for children between 1-5 years old.
5. Currently filming its sixth season, this is the official LEGO reality show.
6. A large molded piece that resembles a rock.
8. An online marketplace, recently purchased by The LEGO Group itself, where you can find old & new sets, minifigures and parts
14. The designer of the LEGO "headlight brick".
15. The material LEGO bricks are made of, abbreviated.
16. Released in 1977, this theme's sets use parts like gears, axles and connectors to create more mechanical models.
17. The town in Denmark where the headquarters of The LEGO Group is located.

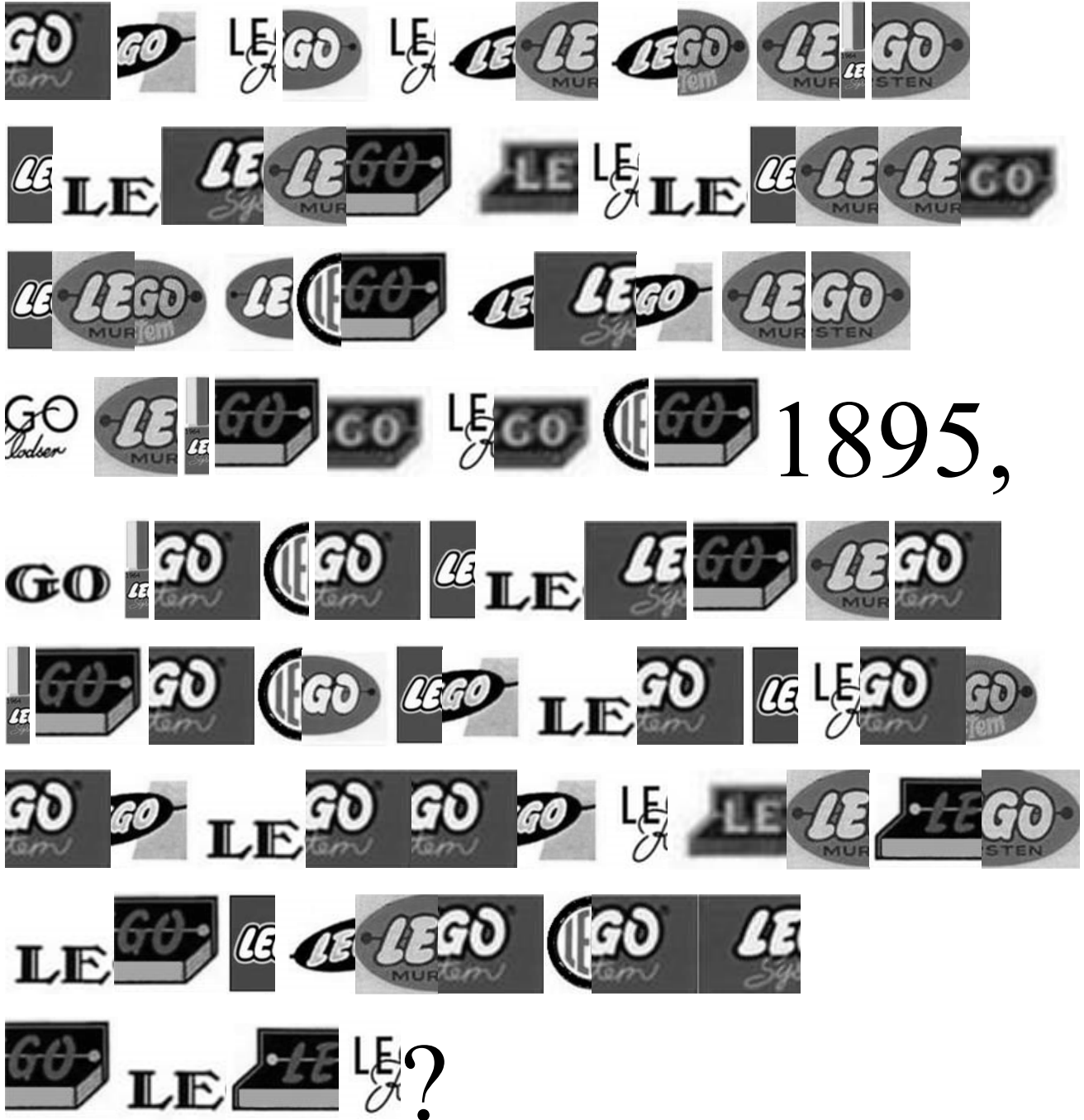
Down

1. Former LEGO theme created for children between ages 1-3.
3. Danish phrase meaning 'play well.'
4. The founder of The LEGO Group.
7. Fan clubs created by LEGO enthusiasts from a particular geographical location that hold certain activities regularly.
9. The name of LEGO Ideas during its beta phase.
10. A person who acts as a liaison to the LEGO Group.
11. A theme that lived between 1979-1989 and was meant for children aged 3-7, to be a transition theme between DUPLO and System.
12. LEGO is the world's largest manufacturer (by units) of these common items.
13. The generic name for each unique LEGO part.

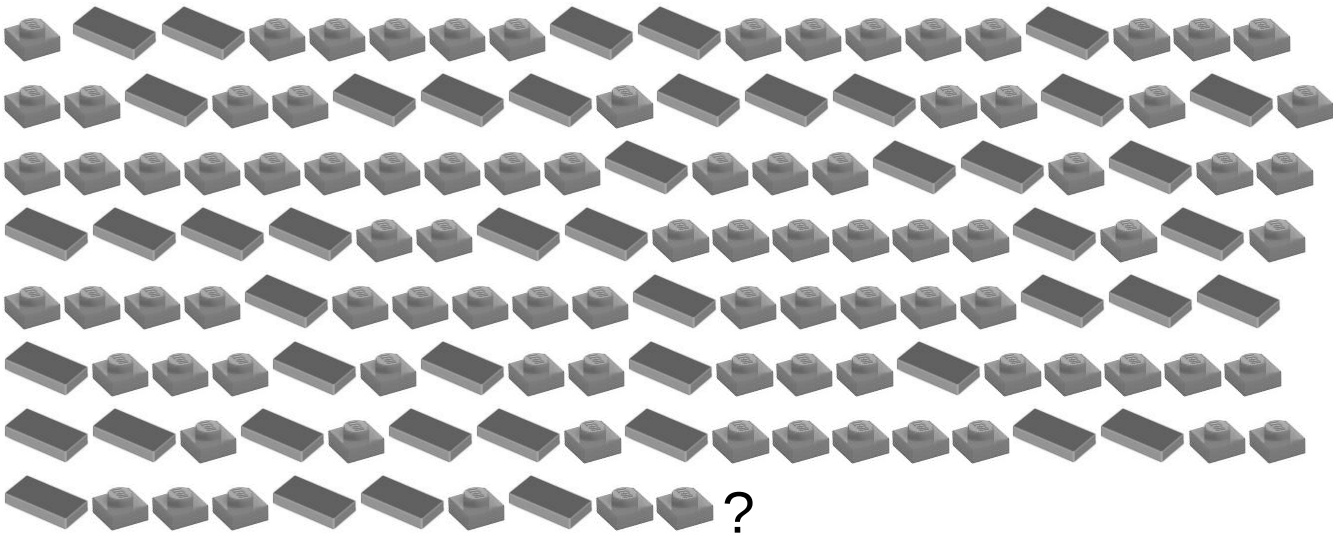
III. (9 points for each decoded question, 3 points for each answer)

For each, decode the question and write the question on the answer sheet for 9 points. Then, if you can answer the question, write the answer in the indicated blank for 3 points.

1.

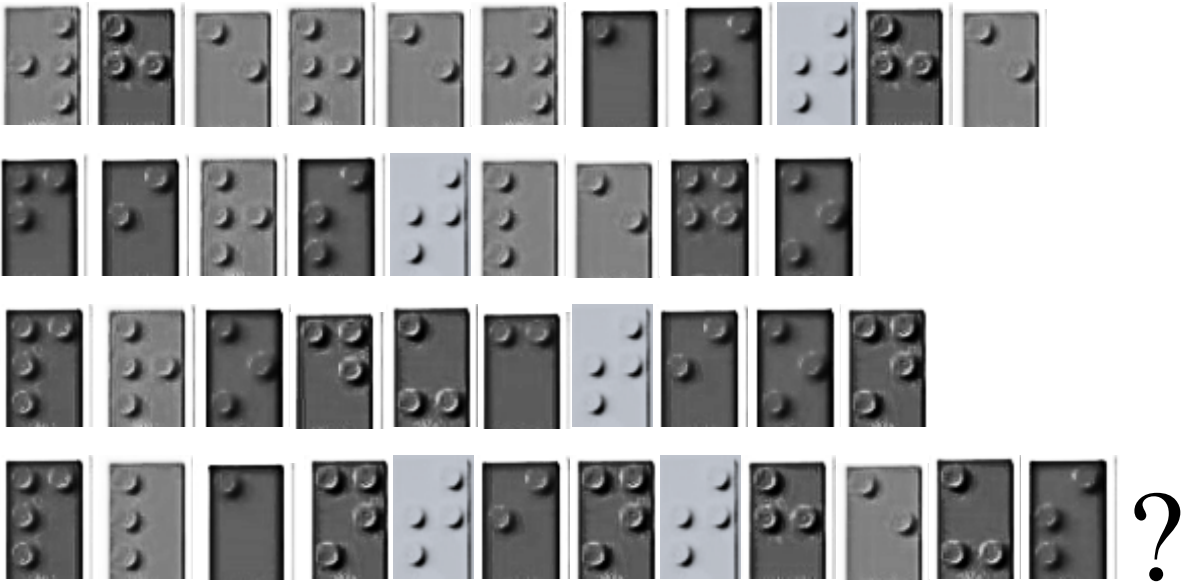


2.

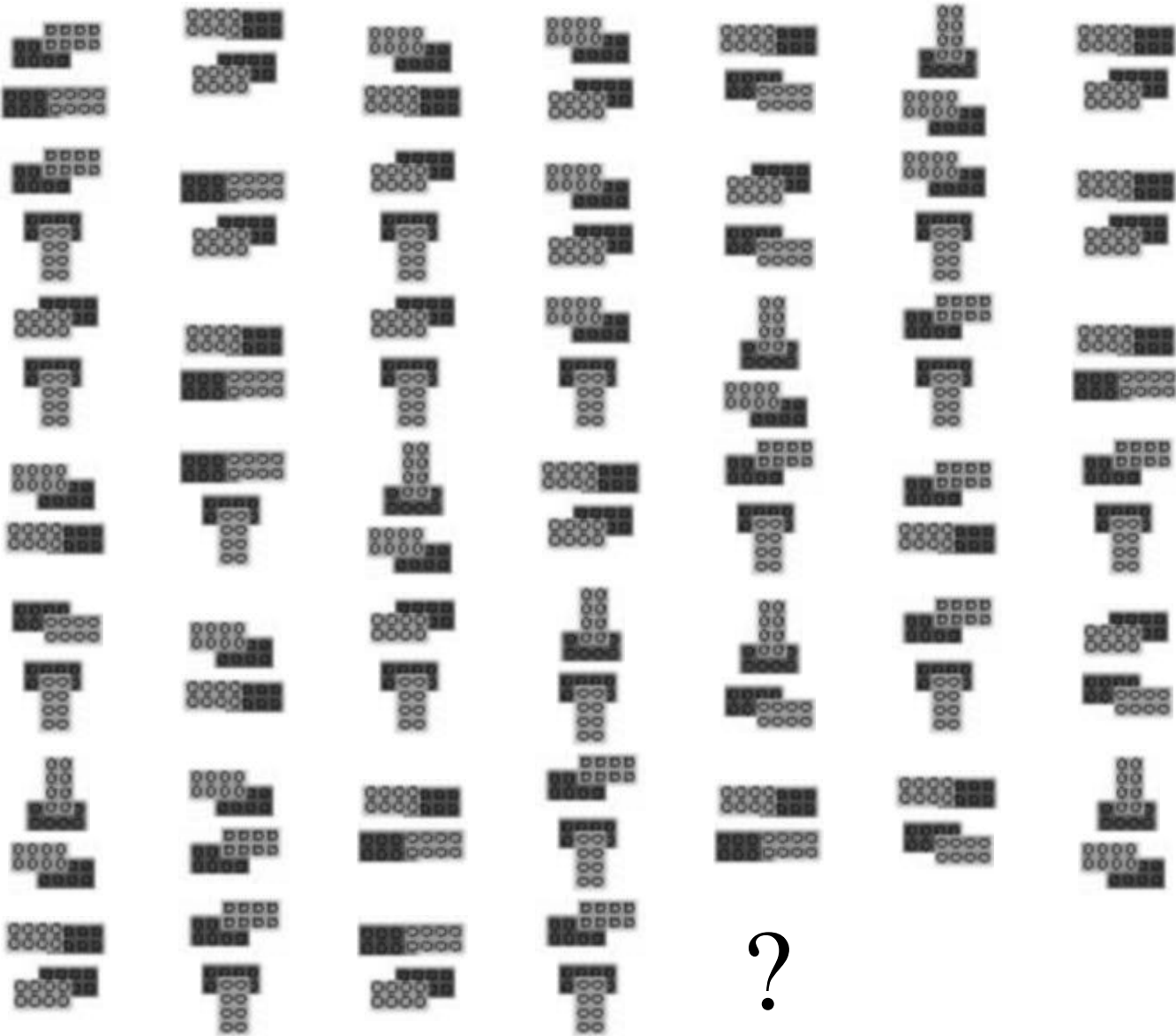


3	4	2	1	2	3	1	4	1	2	2	2	1	3	4	1	4	1	2	3	4	2	2	3	3	2	3	4	2	4
4	1	4	1	4	2	3	2	2	4	4	1	1	4	1	2	1	1	2	3	1	3	4	4	2	4	1			

3.



4.



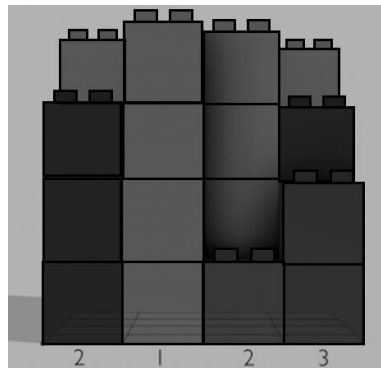
IV. LEGOscapers

Aaron has a pile of LEGO bricks in different colors, and wishes to fill a $N \times N$ grid with stacks of them 1 to N high, such that each height appears only once in every row and column. Once he has done this, he records clues on the edges indicating how many different stacks of bricks are visible from that direction, with taller stacks blocking shorter ones.

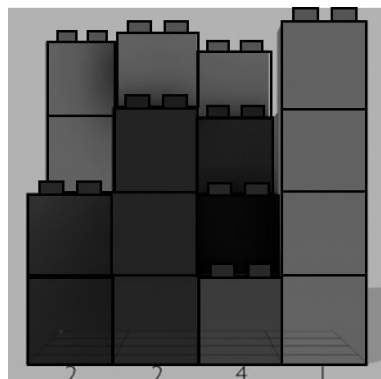
For example, let's say $N = 4$ and Aaron provides the following clues:

	2	3	1	3	
2					2
1					3
2					2
3					1
	2	2	4	1	

These clues came about by viewing the stacks of LEGO bricks from different sides. When viewed from the west, the stacks of LEGO bricks look like this:



Whereas from the south the stacks of bricks look like:



Using logic, one can solve the puzzle by putting numbers in empty boxes indicating the heights of the stacks.
So the solution to the example would be:

	2	3	1	3	
2	3	1	4	2	2
1	4	2	3	1	3
2	1	4	2	3	2
3	2	3	1	4	1
	2	2	4	1	

Solve each of the following N = 4 puzzles. Sometimes, the height of a stack is provided to help solve the overall problem, or some clues are omitted. (6 points each)

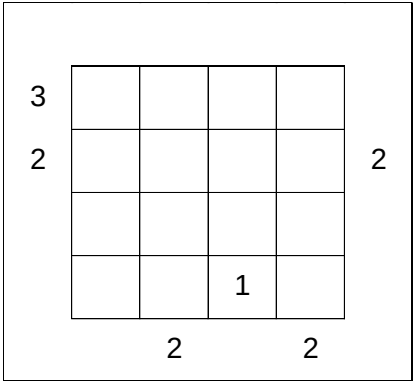
1.

	1	2	2	3	
1					3
3					2
2					1
2					3
	3	1	2	2	

2.

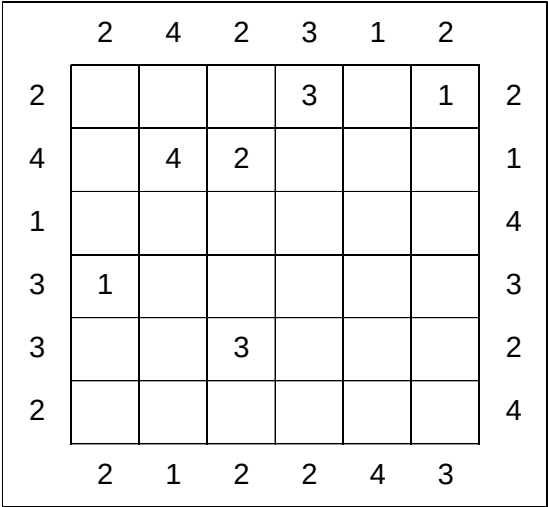
	2				
2					
1					
					2
					1
	2				

3.



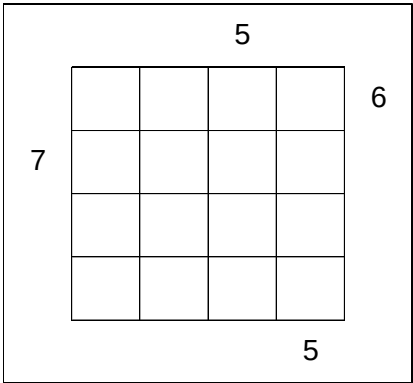
Now that you’ve mastered N = 4, Aaron would like you to solve the following N = 6 puzzle (8 points):

4.

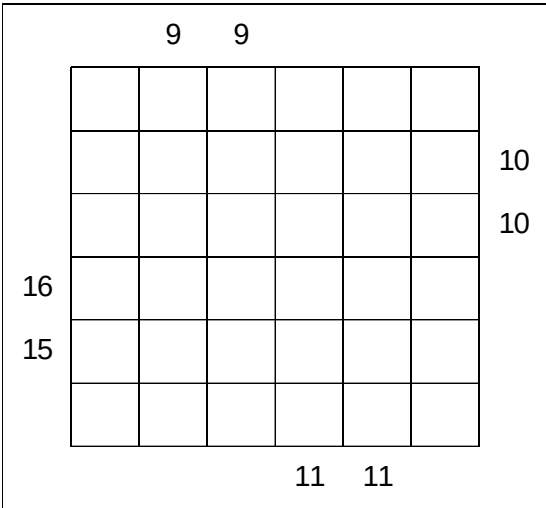


Daniel wants to play the same game as Aaron, but gets the details wrong and accidentally puts the sum of the number of bricks in the visible stacks as clues, rather than the number of visible stacks. But, the puzzles are still solvable! Solve Daniel’s puzzles:

5. (6 points)

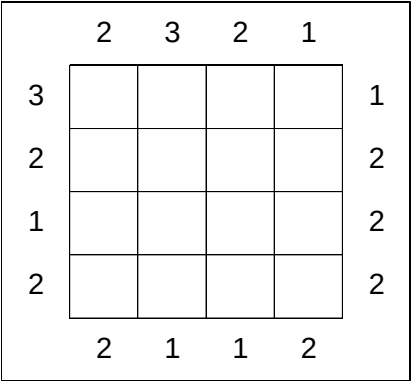


6. (8 points)



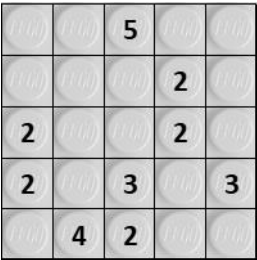
Among Aaron’s LEGO bricks are CLEAR bricks that allow you to see the stack behind them regardless of how tall it is. If Aaron puts one stack of clear bricks in each row and column, each of different heights, then what is the solution to this puzzle? (8 points)

7.

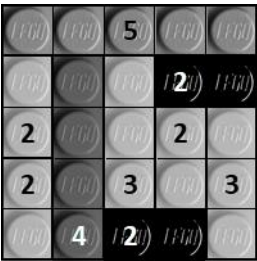


V. ShikakuLEGO

Katie has larger LEGO plates that she wishes to cover with smaller plates. Once she has done so, she challenges Liz to recreate her work by indicating the number of studs on each smaller plate on one of the studs the smaller plate attached to. For example, if the larger LEGO plate to be covered were 5x5, then she might provide the following:



This would have a solution



Which would be put on an answer sheet by bolding the lines and removing the stud picture like this:

		5		
			2	
2			2	
2		3		3
	4	2		

Solve the following six 10x10 versions of this puzzle. Assume LEGO plates of any dimensions are available, even if you believe pieces of that size do not currently exist.

1. (7 points)

[illegible]

			15						
					10				
									10
		9						14	
18					10			6	
			4				4		

[illegible]

4. (8 points)

							5		
	8								
4									
			12						
				6	16				
			12						
								24	
			4	4		5			

5. (9 points)

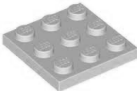
3								16	
	3								
	4	9			3			9	3
		3			3				4
	2			3		3			6
				10		6			
		7						3	

6. (9 points)

					5				
								4	
						4		4	
				8					
6				24				2	
		8			6			4	5
3									
			4				3	2	
		8							

VI. LEGO Hunting (3 points each, 6 points for final answer)

Liam loves LEGO. When he's out in the world, everything reminds him of LEGO.

1.	thing according To shakespeare.	
2.	Urban unit, jennifer lopez's favorite.	
3.	math involving Remainders used in number theory.	
4.	a Matched pair.	
5.	eights, for Ice skaters or cars.	
6.	jethro tull's Measurement of thickness.	
7.	entirely Cover the plane.	
8.	dinner Is served on this.	
9.	Not the apprentice, maybe a key.	
10.	detailed Information telling how something should be done, operated, or assembled.	
11.	misspelled Making fun.	
12.	Wooden house drywall holder.	

When Liam thinks of any other toy, LEGO always wins.