

I. LEGO Math & Bash

1. The fact that it's the fourth and fifth brick is not particularly relevant; we can just treat them as the first and second brick. There is a $\frac{4}{10}$ probability that the first brick is yellow and a $\frac{3}{9}$ probability that the second brick is red given that the first brick is yellow.

Thus the answer is $\frac{3}{9} * \frac{4}{10} = \boxed{\frac{2}{15}}$.

2. If we count not wearing a certain accessory as a possibility, there are 5 possibilities for weapons, 4 for shields, and 6 for capes, for a total of $5 * 4 * 6 = 120$ possibilities. However, this counts the case in which the knight wears none of the accessories, which is not allowed, so the answer is $120 - 1 = \boxed{119}$.

3. There are five options for the head, four options for the torso, and three options for the legs so that none of them are from a minifigure already chosen. So the answer is $\frac{5(4)(3)}{5^3} = \boxed{\frac{12}{25}}$

4. Let ABCDE be the order of the heads chosen for the new five minifigures. Then for no torso to match a head, the ordering of the torsos must be a derangement of the original order. The number of derangements of five elements is $5! \left(\frac{1}{0!} - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} \right) = 44$. So the probability of this occurring is $\frac{44}{5!}$. The leg ordering must also be a derangement of the head order, but it must also be a derangement of the torso ordering. Of the 44 possible torso derangements, 20 have 12 derangements among the 44 derangements of the head order that are also derangements of themselves, and 24 have 13 such derangements. This can be done by casework, among other ways.

Therefore the final answer is $\frac{44}{5!} \left(\frac{\frac{20(12)+24(13)}{44}}{5!} \right) = \boxed{\frac{23}{600}}$

5. a. This conic is rotated by 45° since $\cot(2\theta) = \frac{A-C}{B} = 0$ and it is an ellipse since $B^2 - 4AC < 0$. Applying the transformation $x' = \frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}}$; $y' = \frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}}$ yields $9x'^2 - 18x' + y'^2 + 2y' = 719 \rightarrow \frac{(x'-1)^2}{81} + \frac{(y'+1)^2}{729} = 1 \rightarrow$ the area is $\pi(9)(27) \approx 763.4$. So $[764.4] = \boxed{765}$

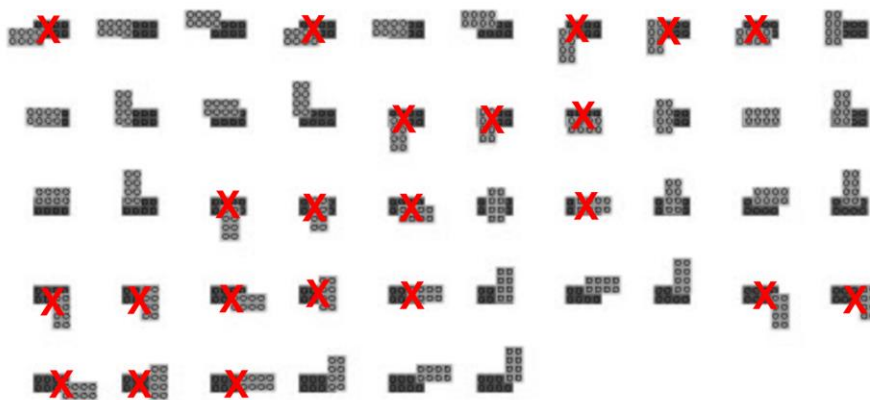
b. The margin of error will be $0.0966 = 1.96 \sqrt{\frac{p(1-p)}{n}}$. This is maximized at $p = \frac{1}{2}$, so $0.0966 = \frac{0.98}{\sqrt{n}} \rightarrow \sqrt{n} = \frac{0.98}{0.0966} \approx 10.14 \rightarrow n \approx 102.8 \rightarrow n = \boxed{103}$

c.
$$\int_0^{\frac{\pi}{2}} \cos^n(x) \sin^5(x) dx = \frac{1}{2} \frac{\left(\frac{n-1}{2}\right)! \left(\frac{5-1}{2}\right)!}{\left(\frac{n-1}{2} + \frac{5-1}{2} + 1\right)!} = \frac{1}{2} \frac{\left(\frac{n-1}{2}\right)! (2)!}{\left(\frac{n-1}{2} + 3\right)!} = \frac{1}{96702120} \text{ by Beta}$$

functions. The denominator factors as $96702120 = 460 * 459 * 458$ so $\frac{n-1}{2} = 457 \rightarrow n = \boxed{915}$.

Final Answer: Considering the much larger number of connection points on towers below height six, the larger of the two possibilities is correct: $\boxed{915,103,765}$

6. There are 24 ways to do this, and here they are:



Duplicates by rotation are crossed out. This occurs because outside of the two invariant stacks under 180° rotation, the remainder each have duplicates. So we get $2 + \frac{44}{2} = \boxed{24}$.

7. To build a tower with six bricks, one can choose among the 46 positions from the previous problem five times. Again, most of the configurations thus obtained have doubles. There are 2^5 symmetric configurations, but the remaining $46^5 - 2^5$ configurations are counted twice. We get $2^5 + \frac{46^5 - 2^5}{2} = \boxed{102,981,504}$

8. The area covered by one 1×2 brick is 15 units^2 . The area covered by the slope brick is $2.5^2 + 0.5 * 5 + \frac{1}{2} (2.5)^2 = 11.875$ or $11\frac{7}{8}$ or $\frac{95}{8}$. Since $\frac{95}{8} * 24 = 285 = 19 * 15$, we are free to cover areas greater than this number and divisible by 15 with 1×2 bricks without needing to worry about sloped bricks.

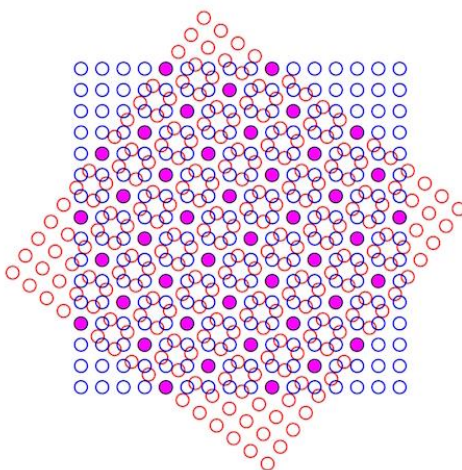
a.
$$\int_0^{50} 2500 - x^2 dx = \left[2500x - \frac{1}{3} x^3 \right]_0^{50} = \frac{250000}{3} \text{ or } 83333.\bar{3}.$$
 A good place to start is $83333.\bar{3} = 83325 + 8.\bar{3} = 5555 * 15 + 8.\bar{3}$. Adding one additional 1×2 brick would make $5556 * 15 = 83340 = 83333.\bar{3} - 6.\bar{6}$. The difference between 15 and $11\frac{7}{8}$

is $3\frac{1}{8} = 3.125$, so changing two 1×2 bricks for slopes would give us $5554 * 15 + 2 * (11\frac{7}{8}) = \boxed{83333.75} = 83333.\bar{3} + 0.41\bar{6}$. No further changes could get closer.

b. $\int_0^{20} 10 \sin\left(\frac{\pi x}{10}\right) dx = \frac{100}{\pi} \left[-\cos\left(\frac{\pi x}{10}\right) \right]_0^{10} = \frac{200}{\pi} \approx 63.7 = 4 * 15 + 3.7$. Adding another 1×2 brick gives us $5 * 15 = 75 = 63.7 + 11.3$. Since the difference between bricks and slopes is 3.125, changing four of the bricks to slopes would yield $1 * 15 + 4 * 11.875 = \boxed{62.5} = 63.7 - 1.2$. This is the best we can do.

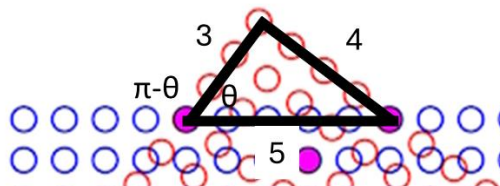
9.

a. The orientation described is:



There are $\boxed{43}$ studs aligned.

b. We see that the plates form a 3-4-5 right triangle:



$$\text{So } \tan(\pi - \theta) = \cot(\theta) = \boxed{\frac{3}{4}}$$

10. As this is based on piece counts, and the function gives the number of digits in the piece count, this is an estimation question. The answers are:

- | | |
|------|------|
| a. 5 | b. 5 |
| c. 4 | d. 4 |

- | | | | |
|----|---|----|---|
| e. | 6 | f. | 7 |
| g. | 3 | h. | 2 |

II. LEGO x Words

Across

- 2. DUPLO
- 5. LEGO Masters
- 6. Big Ugly Rock Piece
- 8. Bricklink
- 14. Erling Dideriksen
- 15. ABS
- 16. TECHNIC
- 17. Billund

Down

- 1. QUATRO
- 3. Leg Godt
- 4. Ole Kirk Christiansen
- 7. LEGO User Group
- 9. Cuusoo
- 10. Ambassador
- 11. Fabuland
- 12. Tires
- 13. Element

III. (LEGOCODES in Wingdings)

- 1. The code is a cryptogram, where each letter is half of a LEGO logo from history. Logos are assigned in historical order with “a” as the LE from the oldest logo etc. The question reads:

THE LEGO GROUP WAS ONCE A WOODWORKING SHOP FOUNDED IN 1895, BUT IT WAS NOT UNTIL WHAT YEAR THAT THE COMPANY GOT ITS NAME?

Answer: 1934

2. This is Morse Code, with the numbers below giving the length of each letter. The question reads:

WHAT IS THE NAME OF THE ISLAND ON WHICH THE BIONICLE THEME TAKES PLACE?

Answer: Mata Nui

3. This is Braille. The question reads:

WHERE WAS THE FIRST LEGO PRODUCTION PLANT IN THE US?

Answer: Enfield, CT

4. This is a variation on Semaphore, with the directions of the bricks vertically in pairs indicating the two compass directions. The question is:

WHO IS THE MAIN CHARACTER OF THE LEGO ADVENTURERS THEME?

Answer: Johnny Thunder

IV. LEGOscapers

1.

	1	2	2	3	
1	4	3	1	2	3
3	1	2	4	3	2
2	3	1	2	4	1
2	2	4	3	1	3
	3	1	2	2	

2.

	2				
2	3	2	4	1	
1	4	1	3	2	
	1	4	2	3	2
	2	3	1	4	1
	2				

3.

3	1	3	2	4	
2	3	1	4	2	2
	2	4	3	1	
	4	2	1	3	
		2		2	

4.

	2	4	2	3	1	2	
2	5	2	4	3	6	1	2
4	3	4	2	5	1	6	1
1	6	5	1	4	2	3	4
3	1	3	6	2	5	4	3
3	2	1	3	6	4	5	2
2	4	6	5	1	3	2	4
	2	1	2	2	4	3	

5.

		5			
7	3	4	1	2	6
	1	2	4	3	
	2	1	3	4	
	4	3	2	1	
			5		

6.

		9	9			
	6	3	1	5	4	2
	5	6	2	3	1	4
	4	5	6	2	3	1
16	1	4	5	6	2	3
15	2	1	3	4	6	5
	3	2	4	1	5	6
			11	11		

7.

		2	3	2	1	
3	3	1	2	4	1	
2	1	3	4	2	2	
1	4	2	1	3	2	
2	2	4	3	1	2	
	2	1	1	2		

V. ShikakuLEGO

1.

			6				4		
						12			
			12						
		6					3		
	18			9					
				15					
							10		
									5

2.

			15						
				10					
								10	
		9						14	
18					10			6	
			4				4		

3.

						18	12		
				10					
12			12						
							10		
4									
	12					10			

4.

						5			
	8								
4									
			12						
				6	16				
			12						
								24	
			4	4		5			

5.

3								16	
	3								
	4	9			3			9	3
		3			3				4
	2			3		3			6
				10		6			
		7						3	

6.

						5			
								4	
							4		4
					8				
6				24				2	
		8			6			4	5
3									
			4				3	2	
		8							

VI. LEGO Hunting

Each is a LEGO-themed word with a single word capitalized, and the number of words in the description corresponds to the number of letters in the answer. Below are the answers, with the letters associated with the capitalized words also capitalized:

1. plAy
2. Block
3. moDular
4. sEt
5. fiGure
6. brIck
7. tIle
8. pLate
9. Master
10. iNstruction
11. mOc
12. Stud

Final Answer:

The LEGO piece next to each clue vary by number of visible studs. Below are the visible studs for each piece.

Letter:	A	B	D	E	G	I	I	L	M	N	O	S
#VisibleStuds:	11	9	8	2	3	6	10	1	5	7	4	12

Using this as a permutation ordering, we get:

LEGOMINDBIAS

Which matches the final clue as well.