



Mental Math

Test #602

Name: _____

ID Number: _____

School: _____

Division (circle one):

Mu Alpha Theta Sponsor

- _____ 1. Evaluate: $\frac{7!}{3!}$
- _____ 2. Evaluate: $\sum_{x=0}^4 (x^2 + 2x + 1)$
- _____ 3. 2026 in base 10 equals what number in base 5?
- _____ 4. Find the sum of the coefficients in the expansion of $(2x^3 + 2x + 6)^2$.
- _____ 5. In the prime factorization of $\binom{45}{20}$, how many times does the factor 3 appear?
- _____ 6. Find the largest positive integer n that solves $(1 + 2 + 3 + \dots + n)^2 \leq 2026$.
- _____ 7. Find the sum of all integers x in the interval $[1, 10]$ that solve the equivalence $x^2 - 5x + 6 \equiv 0 \pmod{7}$.
- _____ 8. A parabola's equation is $y = ax^2 + bx + c$, with a , b , and c real numbers, and it passes through the points $(0, 1)$, $(1, 4)$, and $(2, 9)$. Find the value of $a - b + c$.
- _____ 9. Evaluate: $\begin{vmatrix} 2 & 0 & 8 \\ 2 & 9 & 5 \\ 1 & 0 & 4 \end{vmatrix}$
- _____ 10. What is the expected number of cards that must be flipped until the first King appears in a standard deck of 52 cards? Write your answer as a decimal.
- _____ 11. Find the constant term in the expansion of $\left(x^2 + \frac{1}{x} + 1\right)^6$.
- _____ 12. A sequence with a cubic polynomial rule begins with the numbers 1, 2, 5, 11, 21, 36. What is the next number in the sequence?
- _____ 13. How many solutions (a, b, c) , where a , b , and c are nonnegative integers, satisfy the equation $a + b + c = 6$?
- _____ 14. Todd has x marbles. When he divides them equally among 2, 3, 4, 5, 6, or 7 friends, he has one marble left over. What is the smallest integral value of x greater than 1?
- _____ 15. Find the area enclosed by the polygon with consecutive vertices at the points $(2, 2)$, $(3, 0)$, $(6, 1)$, and $(5, 5)$.
- _____ 16. What is the degree measure of the smaller angle between the hour and minute hands of a clock at 10:40?
- _____ 17. Find n if $\sum_{x=0}^n (2x + 1) = 2025$.
- _____ 18. What is the maximum value of the expression $4(\sin x)(\cos x)$?
- _____ 19. If $P(x) = x^4 - 8x^3 + 9$ and $Q(x) = P(x + 1)$, find the sum of the complex roots of $Q(x)$.
- _____ 20. A disease affects 20% of the population. A test detects the disease 75% of the time when it is actually present, but it also detects the disease 25% of the time when the disease is not actually present. If someone tests positive for the disease, what is the probability that they actually have the disease?
- _____ 21. Find the sum of the mean, median, and range of the data set $\{20, 2, 62, 0, 26\}$.
- _____ 22. What is the largest positive integer that divides $n^3 - n$ for all positive integers n ?
- _____ 23. If $\frac{3}{7}$ is written as a decimal in the form $0.\overline{abcdef}$, find $a + b + c + d + e + f$.
- _____ 24. A box contains 3 yellow balls, 4 green balls, and 5 red balls. Merry randomly draws two balls from the box without replacement. What is the probability that her first drawn ball is yellow and her second drawn ball is red?

- _____ 25. A box contains 3 yellow balls, 4 green balls, and 5 red balls. How many balls does Merry need to draw from the box in order to guarantee 4 balls of the same color have been drawn?
- _____ 26. A convex polyhedron has 12 hexagonal faces. How many vertices does the polyhedron have?
- _____ 27. Calculate the product of the least common multiple and the greatest common divisor of 12 and 15.
- _____ 28. Starting from the floor, Jack is climbing an 8-step staircase. He can take either 1 or 2 steps at a time, but he may not step on the 3rd step. In how many distinct ways can he get to the 8th step?
- _____ 29. Use $\log 2 \approx 0.301$ to estimate the value of $\log 80$.
- _____ 30. Evaluate: $\sum_{k=1}^{50} \left(\frac{1}{k^2 + k} \right)$
- _____ 31. Two chords of a circle intersect perpendicularly. One chord is cut into two equal pieces of length 12 apiece. The other chord is cut into pieces of lengths x and $x+7$. Find the value of x .
- _____ 32. Paul and Will arrive at the mall at random times from 12 pm until 1 pm, independent of each other. Once he arrives, Paul will wait for Will for 15 minutes, then he will leave. Once he arrives, Will will wait for Paul for 30 minutes, then he will leave. What is the probability that they meet at the mall during this time?
- _____ 33. What is the greatest possible value of the greatest common divisor of $n+4$ and $n^2 - 2n - 1$?
- _____ 34. Find the area of the region described by the inequality $|x| + |y| \leq 2$.
- _____ 35. Find x if $x^{x^{x^{x^{\dots}}}} = 2$.
- _____ 36. Let $f(x) = x^3 + bx + c$, where b and c are real numbers. If $f(2+i) = 0$, find the value of c .
- _____ 37. If x is a standard position angle whose terminal side is in the second quadrant, and if $2\sin x + 3\cos x = 0$, find the value of $\sin x$.
- _____ 38. Mary, Sarah, and Allison are all different ages. If Sarah is younger than Mary, what is the probability that Allison is younger than Sarah? Any one girl is equally likely to be older or younger than any other girl.
- _____ 39. Simplify: $\frac{2^{x+2} \cdot 4^{x-1}}{8^x}$
- _____ 40. Let $a = 1, b = 2, c = 3, \dots, z = 26$, assigning to each letter of the English alphabet its corresponding alphabetical position. Evaluate the product $(a-b)(c-d)(e-f)\dots(y-z)$, where all factors are a letter in an odd alphabetical position minus the letter immediately after it, and all letters in an odd alphabetical position are represented.