

1. 840

2. 55

3. 31101

4. 100

5. 3

6. 9

7. 24

8. 0

9. 0

10. 10.6

11. 76

12. 57

13. 28

14. 421

15. 11

16. 80 degrees

17. 44

18. 2

19. 4

20. $\frac{3}{7}$

21. 104

22. 6

23. 27

24. $\frac{5}{44}$

25. 10

26. 26

27. 180

28. 10

29. 1.903

30. $\frac{50}{51}$

31. 9

32. $\frac{19}{32}$

33. 23

34. 8

35. $\sqrt{2}$

36. 20

37. $\frac{3\sqrt{13}}{13}$

38. $\frac{1}{3}$

39. 1

40. -1

1. $\frac{7!}{3!} = 7 \cdot 5! = 7 \cdot 120 = 840$
2. $\sum_{x=0}^4 (x^2 + 2x + 1) = 1^2 + 2^2 + \dots + 5^2$
 $= \frac{5 \cdot 6 \cdot 11}{6} = 55$
3. $2026 = 3 \cdot 5^4 + 1 \cdot 5^3 + 1 \cdot 5^2 + 0 \cdot 5 + 1$,
 so the number is 31101.
4. To get the sum of the coefficients,
 plug in $x = 1$: $(2 + 2 + 6)^2 = 10^2$
 $= 100$
5. $\binom{45}{20} = \frac{45 \cdot 44 \cdot \dots \cdot 26}{20!}$. There are 11
 factors of 3 in the numerator and 8
 factors of 3 in the denominator, so
 the end result contains 3 factors of
 3.
6. Since $45^2 = 2025$, we have
 $1 + 2 + 3 + \dots + n \leq 45$, and equality
 occurs when $n = 9$.
7. $(x - 2)(x - 3) \equiv 0 \pmod{7}$, so
 because 7 is prime, x must be 2 or
 3 more than a multiple of 7, which
 is satisfied by 2, 3, 9, and 10.
 $2 + 3 + 9 + 10 = 24$
8. The parabola is $y = (x + 1)^2$, and
 $a - b + c$ is the result when -1 is
 plugged in, so $(-1 + 1)^2 = 0$.
9. Row 1 is twice row 3, so the
 determinant is 0.
10. The four Kings partition the deck
 into 5 components with average
 size $\frac{52 - 4}{5} = 9.6$, so we expect to
 draw 9.6 cards before the first
 King, making the average number
 10.6 cards.
11. The individual terms do not
 contribute to the coefficient since
 they are made of ones only, the
 coefficient would be (factorials in
 the denominator represent powers
 of each term, from left to right):
 $\frac{6!}{2!4!0!} + \frac{6!}{1!2!3!} + \frac{6!}{0!0!6!} = 15 + 60$
 $+ 1 = 76$.

12. Using finite differences, because this is a cubic rule, the differences are going up by the triangular numbers, so the next number is $36 + 21 = 57$.

13. Using the stars and bars method, the number of ways to do this is $\binom{3+6-1}{6} = \binom{8}{6} = \frac{8 \cdot 7}{2 \cdot 1} = 28$.

14. The smallest number greater than 1 is 1 more than the least common multiple of 2, 3, 4, 5, 6, and 7, which is $1 + 420 = 421$.

15. Using the Shoelace method, the area is $\frac{1}{2} |2 \cdot 3 + 0 \cdot 6 + 1 \cdot 5 + 5 \cdot 2 - 2 \cdot 0 - 3 \cdot 1 - 6 \cdot 5 - 5 \cdot 2| = 11$.

16. The angle is $\frac{1}{2} |60 \cdot 10 - 11 \cdot 40| = 80$ degrees.

17. $\sum_{x=0}^n (2x+1)$ is the sum of the first $n+1$ odd positive integers, which equals $(n+1)^2$. Therefore, $(n+1)^2 = 2025 \Rightarrow n = 44$.

18. $4(\sin x)(\cos x) = 2\sin 2x$, so the maximum value is 2.

19. The four roots are all distinct, and their sum is 8 by Vieta's formulas. The roots for Q are each 1 less than that of P , so the sum of those roots is $8 - 4 \cdot 1 = 4$.

20. A person who tests positive either has it $\left(\frac{1}{5} \cdot \frac{3}{4} = \frac{3}{20}\right)$ or doesn't have it $\left(\frac{4}{5} \cdot \frac{1}{4} = \frac{4}{20}\right)$, so by Bayes' theorem, this probability is

$$\frac{\frac{3}{20}}{\frac{3}{20} + \frac{4}{20}} = \frac{3}{7}.$$

21. The mean is $\frac{20+2+62+0+26}{5} = 22$, the median is 20, and the range is $62 - 0 = 62$, so the sum of these numbers is $22 + 20 + 62 = 104$.

22. The least positive value of the expression is 6, so the answer can't be more than 6. However, $n^3 - n$

$= (n-1)(n)(n+1)$, which is the product of 3 consecutive integers.

Within any set of 3 consecutive integers, one number is divisible by 3 and at least one number is divisible by 2, so this expression is divisible by 6, no matter what positive integer n is.

$$23. \frac{3}{7} = 0.\overline{428571}, \text{ so } .4 + 2 + 8 + 5 + 7 + 1 = 27$$

$$24. \frac{3 \cdot 5}{12 \cdot 11} = \frac{5}{44}$$

25. In the worst-case scenario, Merry's first drawn balls, in some order, are all 3 yellow balls, 3 green balls, and 3 red balls. The next drawn ball would either be green or red, ensuring 4 balls drawn of that color, so it takes 10 draws.

26. Each face has six edges, but each edge is on two different faces, so

$$\text{there are } \frac{6 \cdot 12}{2} = 36 \text{ total edges.}$$

$$\text{Using Euler's formula, } V - 36 + 12 = 2 \Rightarrow V = 26.$$

27. The product of the LCM and GCD of two numbers equals the product of the two numbers, so this product is $12 \cdot 15 = 180$.

28. Let F_n be the number of ways to get to the n th step. The recursive definition is $F_n = F_{n-1} + F_{n-2}$ for $n \geq 4$, with $F_1 = 1$, $F_2 = 2$, and $F_3 = 0$. Therefore, the next five values of this sequence are 2, 2, 4, 6, 10, making 10 ways to get to the 8th step.

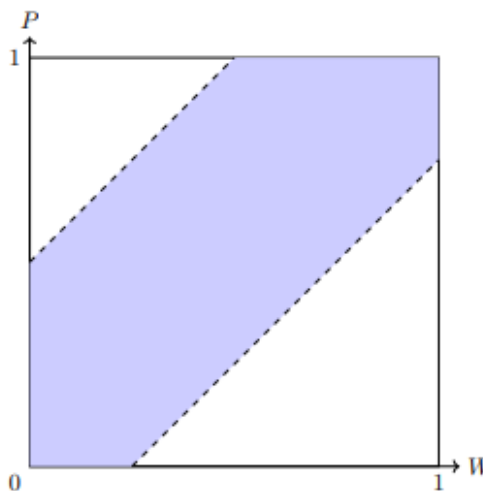
$$29. \log 80 = 3 \log 2 + \log 10 \approx 3(0.301) + 1 = 1.903$$

30. This is a telescoping series, and

$$\sum_{k=1}^{50} \left(\frac{1}{k^2 + k} \right) = \sum_{k=1}^{50} \left(\frac{1}{k} - \frac{1}{k+1} \right)$$

$$= 1 - \frac{1}{51} = \frac{50}{51}.$$

31. $x(x+7) = 12^2 \Rightarrow x = 9$



32.

Draw a 1x1 square, where the values along the edges correspond to the times that they show up.

The shaded part is when they meet up, with the white portions being isosceles right triangles with leg

lengths of $\frac{1}{2}$ and $\frac{3}{4}$. The shaded

region's area, then, is $1 - \frac{1}{2} \left(\left(\frac{1}{2} \right)^2 \right)$

$$+ \left(\frac{3}{4} \right)^2 = \frac{19}{32}.$$

33. $\text{GCD}(n+4, n^2 - 2n - 1) =$

$$\text{GCD}(n(n+4) - (n^2 - 2n - 1), n+4) =$$

$$\text{GCD}(6n+1, n+4) =$$

$$\text{GCD}(6(n+4) - (6n+1), n+4) =$$

$$\text{GCD}(23, n+4), \text{ so the greatest}$$

common divisor is 23, which

occurs when $n = 19$.

34. This is a square region with vertices at the x - and y -intercepts of ± 2 , so the enclosed area is

$$(2\sqrt{2})^2 = 8.$$

35. $2 = x^{x^{x^{x^{\dots}}}} = x^2 \Rightarrow x = \sqrt{2}$ (since x must be positive)

36. By Vieta's formulas, the sum of all three roots is 0, and since $2 - i$ must also be a root, the third root must be -4 . Therefore, again by

$$\text{Vieta's formulas, } \frac{-c}{1}$$

$$= -4(2+i)(2-i) = -20 \Rightarrow c = 20.$$

37. Solving this equation yields

$$\tan \theta = -\frac{3}{2}, \text{ so because the angle is}$$

$$\text{in quadrant II, } \sin \theta = \frac{3}{\sqrt{(-2)^2 + 3^2}}$$

$$= \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}.$$

38. There are $3! = 6$ orderings of the girls' birth orders, all of which are equally likely. The orders from youngest to oldest are MSA, MAS, AMS, ASM, SAM, SMA, but in only the last three is Sarah younger than Mary, which was given. Of those three, only the first one has Allison younger than Sarah, so the probability is $\frac{1}{3}$.

$$\begin{aligned} 39. \frac{2^{x+2} \cdot 4^{x-1}}{8^x} &= \frac{2^{x+2} \cdot 2^{2x-2}}{2^{3x}} \\ &= 2^{(x+2)+(2x-2)-3x} = 2^0 = 1 \end{aligned}$$

40. There are 13 factors, each of which equals -1 , so this product is

$$(-1)^{13} = -1.$$