

For all questions, answer E. “NOTA” means none of the above answers is correct.

1. Rectangle $ABCD$ satisfies $AB = 4BC$. Let E be a point on $\overline{AB}$. Find the ratio of the area enclosed by triangle ECD to the area enclosed by rectangle $ABCD$.

- A: $\frac{3}{4}$ B: $\frac{2}{3}$ C: $\frac{1}{2}$ D: $\frac{1}{3}$ E: NOTA

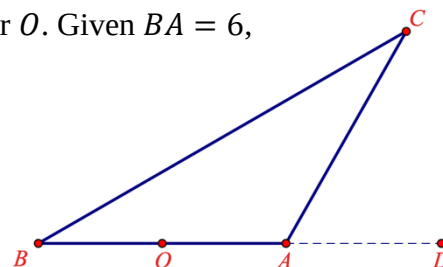
2. A cylinder of radius $4\sqrt{\pi}$ and a cube have the same total surface area. If the height of the cylinder is $2\sqrt{\pi}$, what is the volume of the cube?

- A: $16\pi^3\sqrt{2}$ B: $8\pi^3\sqrt{2}$ C: $8\pi^2$ D: $16\pi^2$ E: NOTA

3. Find the area of the triangular region bounded by $y = 2026x$, $y = 2026 - 2026x$ and $y = 0$.

- A: 2026 B: $\frac{2026}{2}$ C: 1013 D: $\frac{1013}{2}$ E: NOTA

4. Consider the side profile of an oblique circular cone with base center O . Given $BA = 6$, $m\angle CBA = \frac{\pi}{6}$, and $m\angle CAD = \frac{\pi}{3}$. What is the volume of the cone?



- A: $6\pi\sqrt{3}$ B: $9\pi\sqrt{3}$ C: $12\pi\sqrt{3}$ D: $36\pi\sqrt{3}$ E: NOTA

5. $\triangle MAO$ has sides of length $MA = 8$ and $MO = 6$. What is the maximum area enclosed by $\triangle MAO$?

- A: 12 B: 24 C: 36 D: 48 E: NOTA

6. The legs of a right triangle are in the ratio $\sqrt{3}:1$. The perimeter of the triangle is $12 + 4\sqrt{3}$. What is the area enclosed by the triangle?

- A: 8 B: $8\sqrt{3}$ C: 16 D: $16\sqrt{3}$ E: NOTA

7. Consider the function $f(x) = \begin{cases} \frac{x}{2}, & 0 \leq x \leq 6 \\ \sqrt{-x^2 + 12x - 27}, & 6 < x \leq 9 \end{cases}$. Find the volume of the solid generated when the region bounded by the graph of $y = f(x)$ and the x -axis is revolved about $y = 0$.

- A: 18π B: 24π C: 36π D: 42π E: NOTA

8. Find the area of the region in quadrant I above the x -axis and below the curve $y = x^4 e^{-2x}$.

A: $\frac{1}{2}$

B: $\frac{3}{5}$

C: $\frac{2}{3}$

D: $\frac{3}{4}$

E: NOTA

9. A trapezoid has a constant area of 40 square units. One base is always three times the other. The larger base decreases at a rate of 6 units per minute. When the smaller base is 2 units, which of the following describes the rate of change of the trapezoid's height?

A: The height is increasing at a rate of 10 units per minute.

B: The height is increasing at a rate of 5 units per minute.

C: The height is decreasing at a rate of 10 units per minute.

D: The height is decreasing at a rate of 5 units per minute.

E: NOTA

10. The region R is bounded by the curves $y = x^2 + 2x$ and $y = 12 - x^2$. The line $y = k$ divides R into two regions of equal area. What is the value of k ?

A: $\frac{9}{2}$

B: 5

C: $\frac{11}{2}$

D: 6

E: NOTA

11. Which integral represents the volume of the solid formed when the region bounded by the graphs of $y = x^e$, $y = e^x$, $y = 0$, and $x = 0$ is rotated about the line $y = -e$?

A: $\pi \int_0^e ((e^x + e)^2 - (x^e + e)^2) dx$

B: $\pi \int_0^e ((e^x - e)^2 - (x^e - e)^2) dx$

C: $\pi \int_0^e ((x^e + e)^2 - (e^x + e)^2) dx$

D: $\pi \int_0^e (x^e + e^x + 2)^2 dx$

E: NOTA

12. A rock is thrown into a lake. A ripple in the shape of a circle expands such that its radius increases at a rate of $\frac{5}{\pi}$ cm per second. What is the rate of change of the area within the ripple, in square centimeters per second, when the area within the ripple is 25π square centimeters?

A: 5

B: 25

C: 50

D: 100

E: NOTA

13. Rectangle $PQRS$ is drawn so its bottom left vertex P is at the origin and its upper right vertex R lies on the graph of the function $f(x) = e^{-2x}$ (its sides are parallel to the coordinate axes). What is the maximum area enclosed by $PQRS$?

- A: $\frac{1}{e^2}$ B: $\frac{1}{2e}$ C: $\frac{1}{e}$ D: e E: NOTA

14. Find the area of the region bounded above by the curve $y = \frac{1-x^2}{\sqrt{x^2+1}}$ and below by the x -axis.

- A: $3 \ln(1 + \sqrt{2}) - \sqrt{2}$ C: $2 \ln(1 + \sqrt{2}) - 1 + \ln 2$ E: NOTA
 B: $2 \ln(1 + \sqrt{2}) + \sqrt{2}$ D: $2 \ln(1 + \sqrt{2}) - \ln 2$

15. Find the area of the region bounded by the curve $r = 2\sqrt{2} + 2 \cos \theta - 2 \sin \theta$.

- A: 6π B: 8π C: 10π D: 12π E: NOTA

16. You've made it halfway through the test! Sphere S has half the radius of sphere T . What is the ratio of the volume enclosed by S to the volume enclosed by T ?

- A: 1:8 B: 1:4 C: 1:2 D: 2:1 E: NOTA

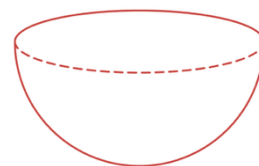
17. Consider the region R bounded by $y = 2\sqrt{\ln(x+1)}$, $y = 0$, and $x = 3$. Create the solid S where each cross-section perpendicular to the x -axis is a regular hexagon with one of its sides of maximum length lying in R . Find the volume of S .

- A: $18\sqrt{3} \ln 3 - 18\sqrt{3}$ C: $24\sqrt{3} \ln 3 - 6\sqrt{3}$ E: NOTA
 B: $24\sqrt{3} \ln 4 - 18\sqrt{3}$ D: $18\sqrt{3} \ln 4 - 18\sqrt{3}$

18. Let n be a positive integer and $R > 0$. What is the interior area bounded by the polar curve $r = R \sin(2n\theta)$?

- A: πR^2 B: $\frac{\pi R^2}{4}$ C: $\frac{\pi R^2}{8n}$ D: $\frac{\pi R^2 n}{2}$ E: NOTA

19. A hemispherical bowl has a radius of 4 inches, with its base laying flat along a surface, as shown. Water is pumped into the bowl at a rate of 4π cubic inches per second. Find the rate of change of the height of water in the bowl (in inches per second) the moment the water reaches a height of 2 inches from the base.



- A: $\frac{1}{6}$ B: $\frac{1}{3}$ C: $\frac{1}{4}$ D: $\frac{1}{2}$ E: NOTA

20. Let $n > 0$ and define $f(x) = e^{-\frac{x^2}{n}}$. When the region above the x -axis and below the graph of $y = f(x)$ is revolved about the y -axis, the resulting region has volume 2026π . Find the value of n .

- A: 4052 B: 2026 C: 1013 D: $\sqrt{2026}$ E: NOTA

21. Which integral represents the surface area when the polar curve given by $r = \cos^2 \theta$ for $0 \leq \theta \leq \frac{\pi}{2}$ is rotated about the y -axis?

- A: $2\pi \int_0^{\frac{\pi}{2}} \cos^4 \theta \sqrt{\cos^2 \theta + 4 \sin^2 \theta} d\theta$ B: $2\pi \int_0^{\frac{\pi}{2}} \cos^3 \theta \sqrt{\cos^2 \theta + 4 \sin^2 \theta} d\theta$
 C: $\pi \int_0^{\frac{\pi}{2}} \cos^4 \theta \sqrt{\cos^2 \theta + 2 \sin^2 \theta} d\theta$ D: $\pi \int_0^{\frac{\pi}{2}} \cos^3 \theta \sqrt{\cos^2 \theta + 2 \sin^2 \theta} d\theta$
 E: NOTA

22. Consider the table below for selected values of x for continuous increasing function $f(x)$:

x	0	2	5	8	10
$f(x)$	$\sqrt{2}$	$\sqrt{3}$	$2\sqrt{2}$	$4\sqrt{3}$	$8\sqrt{2}$

Let K be the signed area (relative to the x -axis—positive if area is above the x -axis, negative if area is below the x -axis) of $f(x)$ on $[0, 10]$. Let L be the left Riemann sum approximation of K using 4 subintervals indicated by the table. Which statement is true?

- A: $L = 8\sqrt{2} + 11\sqrt{3}$ is an overestimate of K .
 B: $L = 8\sqrt{2} + 11\sqrt{3}$ is an underestimate of K .
 C: $L = 22\sqrt{2} + 14\sqrt{3}$ is an overestimate of K .
 D: $L = 22\sqrt{2} + 14\sqrt{3}$ is an underestimate of K .
 E: NOTA

23. Let R be the region in the plane bounded by $f(x) = \sin(2x)$, $x = 0$, $x = \frac{\pi}{4}$, and $y = 0$. Let T represent the Trapezoidal Rule approximation of the area of R with $n = 4$ equal subintervals of $\left[0, \frac{\pi}{4}\right]$. Which of the following is the Trapezoidal Rule error bound for this approximation?

- A: $\frac{\pi^2}{192}$ B: $\frac{\pi^2}{768}$ C: $\frac{\pi^3}{1728}$ D: $\frac{\pi^3}{3072}$ E: NOTA

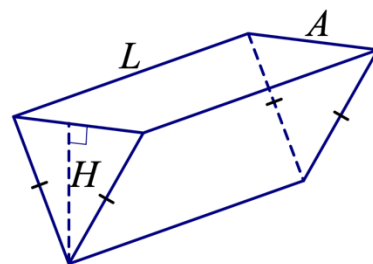
24. Consider the parametrically-defined curve given by $x(t) = t, y(t) = \sqrt{1+t^2}$ for $0 \leq t \leq 1$. The area of the resulting surface when the curve is rotated about the x -axis can be written as $\pi\sqrt{p} + \frac{\pi}{2}\sqrt{q} \ln(\sqrt{r} + \sqrt{s})$ for positive primes p, q, r, s . Find $p + q + r + s$.

- A: 8 B: 9 C: 10 D: 12 E: NOTA

25. A dog is on a leash that is connected to the outside corner of a rectangular shed which measures 6 meters by 12 meters. The dog's leash is L meters long and $12 < L < 18$. If the total area of the region that the dog is able to reach, which is completely flat, is 164π square meters, what is the value of L ?

- A: 12 B: $8\sqrt{3}$ C: 14 D: $12\sqrt{2}$ E: NOTA

26. A trough full of water is in the shape of a triangular prism with dimensions as shown. The density of water is $1000 \frac{\text{kg}}{\text{m}^3}$ and the acceleration due to gravity is $g = 9.8 \frac{\text{m}}{\text{s}^2}$. What is the total work required to pump all the water out to the top of the trough?



A: $\frac{500gALH^2}{3}$

C: $\frac{500gALH^3}{6}$

E: NOTA

B: $\frac{500gALH^3}{3}$

D: $\frac{500gALH^2}{6}$

27. Let $a > 1$. Consider the solid S in coordinate space given by $\left|\frac{x}{a-1}\right| + |a^2y| + \left|\frac{z}{a-1}\right| \leq 1$. Let $V(a)$ denote the volume of S . Find $\lim_{a \rightarrow \infty} V(a)$?

A: $\frac{1}{6}$

B: $\frac{2}{3}$

C: $\frac{4}{3}$

D: 4

E: NOTA

28. Let R be the region in the first quadrant bounded by the x -axis, the graph of $y = \frac{x}{\sqrt{x^2+16}}$, and the vertical line $x = 2k$, where $k > 0$. The value of k increases at a rate of 3 units per minute. What is k when the rate of change of the area with respect to time is $\frac{18}{5}$ square units per minute?

A: $\frac{1}{2}$

B: $\frac{3}{2}$

C: 2

D: 3

E: NOTA

29. Define the following regions in the plane: $R: x^2 + y^2 \leq 2x + 3$, $S: x^2 + y^2 \leq -2x + 3$, $T: x^2 + y^2 \leq 2y + 3$, $U: x^2 + y^2 \leq -2y + 3$. Find the area of $R \cap S \cap T \cap U$.

A: $4\pi - 2 - 16 \arcsin\left(\frac{\sqrt{2}}{4}\right) - 2\sqrt{7}$ C: $4\pi + 2 - 8 \arcsin\left(\frac{\sqrt{2}}{4}\right) - 2\sqrt{7}$ E: NOTA

B: $4\pi + 2 - 16 \arcsin\left(\frac{\sqrt{2}}{4}\right) - 2\sqrt{7}$ D: $4\pi - 2 - 16 \arcsin\left(\frac{\sqrt{2}}{4}\right) - \sqrt{7}$

30. The perimeter of a rectangle is $16\sqrt{2}$. What is the maximum area enclosed by the rectangle?

A: 16 B: 32 C: $32\sqrt{2}$ D: $64\sqrt{2}$ E: NOTA