

Important Instructions for this Test: The answer choice E: NOTA indicates that “None Of These Answers” are correct. Problems are *not* in order of increasing difficulty; feel free to skip around. Good luck, and have fun!

1. Legosi is attempting to compute $\lim_{x \rightarrow \infty} \frac{\cos(x) - x}{x}$. Recognizing the form $\frac{\infty}{\infty}$, he uses L'Hôpital's rule and gets that this limit is equal to $\lim_{x \rightarrow \infty} \frac{-\sin(x) - 1}{1}$, which is simply equal to... nothing. That's weird! What's the value of the original limit? (For the purposes of this test, the acronym 'DNE' means “does not exist,” which includes the case where the limit diverges to infinity.)

A: -1 B: 1 C: 0 D: DNE E: NOTA

2. Does $\int_0^1 \frac{1}{\sqrt{x}} dx$ converge? Does $\int_1^\infty \frac{1}{\sqrt{x}} dx$ converge?

A: yes; yes B: yes; no C: no; yes D: no; no E: NOTA

3. Does $\sum_2^\infty \frac{1}{n(\ln n)^2}$ converge? Does $\int_2^\infty \frac{1}{x(\ln x)^2} dx$ converge?

A: yes; yes B: yes; no C: no; yes D: no; no E: NOTA

4. Consider the curve defined parametrically by $x(t) = \sin(t)$, $y(t) = \cos(t)$ for $0 \leq t < 2\pi$. Which of the following is equal to the slope of the line tangent to this curve at a given value of t (where defined)?

A: $\tan(t)$ B: $\cot(t)$ C: $-\tan(t)$ D: $-\cot(t)$ E: NOTA

5. Evaluate $\int_0^1 x^{2026} \ln(x) dx$.

A: $-\frac{1}{2026}$ B: $-\frac{1}{2027}$ C: $-\frac{1}{2026^2}$ D: $-\frac{1}{2027^2}$ E: NOTA

6. Evaluate $\int_1^2 \frac{1}{x^2 + 4x + 3} dx$.

A: $\frac{\pi}{4}$ B: $\frac{\pi}{2}$ C: $\ln\left(\frac{6}{5}\right)$ D: $\frac{1}{2}\ln\left(\frac{6}{5}\right)$ E: NOTA

7. Evaluate $\int_1^2 \frac{1}{x^2 - 4x + 5} dx$.

A: $\ln\left(\frac{5}{4}\right)$ B: $\frac{1}{3}\ln\left(\frac{5}{4}\right)$ C: $\frac{\pi}{4}$ D: $\frac{\pi}{2}$ E: NOTA

8. The curve defined parametrically by $x(t) = t^2 - 2t$, $y(t) = t^3 - 3t$ has a cusp (a point where, if you trace along the curve as t increases, the curve reverses direction). Find the sum of the rectangular coordinates of this cusp.

A: -3 B: 0 C: 1 D: 4 E: NOTA

9. The curve defined parametrically by $x(t) = t^2 - 2t$, $y(t) = t^3 - 4t$ intersects itself exactly once, forming a single closed region. Find the area of this region.

A: $\frac{8}{15}$ B: $\frac{37}{30}$ C: $\frac{16}{5}$ D: 3 E: NOTA

10. Find the slope of the line tangent to the graph of $r = 2 + \cos(\theta)$ where $\theta = \frac{\pi}{6}$.

- A: $-\frac{1+\sqrt{3}}{4}$ B: $2 - 2\sqrt{3}$ C: $-\frac{4+3\sqrt{3}}{11}$ D: $4 - 3\sqrt{3}$ E: NOTA

11. Find the area enclosed by the graph of $r = 2 + \cos(\theta)$.

- A: 3π B: $\frac{7\pi}{2}$ C: 4π D: $\frac{9\pi}{2}$ E: NOTA

12. Find the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{x^{2n}}{4^n n^2}$.

- A: $[-4,4)$ B: $(-4,4)$ C: $[-2,2)$ D: $(-2,2)$ E: NOTA

13. Evaluate

$$\sum_{n=4}^{\infty} \frac{1}{n^2 - 1}$$

- A: $\frac{7}{12}$ B: $\frac{7}{24}$ C: $\frac{9}{20}$ D: $\frac{9}{40}$ E: NOTA

14. Evaluate

$$\int_0^1 \frac{x^2 + x + 1}{(x^2 + 1)^2} dx.$$

- A: $\frac{\pi+2\ln(2)}{4}$ B: $\frac{\pi+2\ln(2)}{4}$ C: $\frac{\pi+1}{4}$ D: $\frac{\pi+2}{4}$ E: NOTA

15. Compute the volume swept out when the region enclosed by $x^4 + y^2 = 1$ is revolved around the y -axis.

- A: $\frac{\pi}{4}$ B: $\frac{\pi}{8}$ C: $\frac{\pi^2}{2}$ D: $\frac{\pi^2}{4}$ E: NOTA

16. For all $0 \leq x \leq 1$, $\int_0^x e^{\sin^{-1}(t)} dt$ is equal to $f(x)e^{\sin^{-1}(x)} + C$ for some function $f(x)$ and real number C . Evaluate $\left(\int_0^1 f(x) dx\right) + C$.

- A: $\frac{\pi+2}{4}$ B: $\frac{\pi-2}{4}$ C: $\frac{\pi+2}{8}$ D: $\frac{\pi-2}{8}$ E: NOTA

17. Evaluate

$$\int_0^{\infty} e^{-x} \cos^3(x) dx.$$

- A: $\frac{2}{5}$ B: $\frac{1}{5}$ C: $\frac{2}{3}$ D: $\frac{1}{3}$ E: NOTA

18. Compute the 3rd degree Taylor polynomial for $\sqrt{x}$ centered at $x = 1$, and use it to approximate $\sqrt{2}$.

- A: $\frac{11}{8}$ B: $\frac{23}{16}$ C: $\frac{13}{8}$ D: $\frac{27}{16}$ E: NOTA

19. Evaluate

$$\int_0^1 \frac{\tan^{-1}(x)}{(1+x)^2} dx.$$

- A: $\frac{\pi^2}{32}$ B: $\frac{\pi}{8}$ C: $\frac{\ln(2)}{4} + \frac{\pi}{4}$ D: $\frac{\ln(2)}{4}$ E: NOTA

20. Find the surface area of the surface formed by revolving the graph of $y = \sqrt{x}$ between $x = 0$ and $x = 6$ around the x -axis.

- A: $\frac{31\pi}{6}$ B: $\frac{31\pi}{3}$ C: $\frac{62\pi}{3}$ D: $\frac{124\pi}{3}$ E: NOTA

21. Find the length of the polar curve given by $r = 2 + 2 \cos(\theta)$.

- A: 8π B: 4π C: 16 D: 8 E: NOTA

22. Consider the differential equation $y' = x + y$, where y is a function of x . Given that $y(0) = 1$, approximate $y(2)$ using Euler's method with four equal-sized steps.

- A: $\frac{43}{2}$ B: $\frac{27}{4}$ C: $\frac{41}{4}$ D: $\frac{57}{8}$ E: NOTA

23. Let's see how you did! Given $y' = x + y$ with $y(0) = 1$ from the previous problem, find a function $I(x)$ such that $y'I(x) - yI(x)$ looks like a product rule, and then integrate both sides to solve for y exactly (such an $I(x)$ is called an *integrating factor*). Compute $y(2)$.

- A: $e^2 - 3$ B: $2e^2 - 3$ C: -3 D: $e^{-2} + 3$ E: NOTA

24. Evaluate

$$\int_0^{\frac{\pi}{2}} \sin^3(x) \cos^4(x) dx.$$

- A: $-\frac{2}{35}$ B: $\frac{\pi}{24}$ C: $\frac{2}{35}$ D: $-\frac{\pi}{24}$ E: NOTA

25. Evaluate

$$\int_0^{\frac{\pi}{4}} \tan^2(x) \sec(x) dx.$$

- A: $\frac{1}{2}(\sqrt{2} + \ln(1 + \sqrt{2}))$ C: $\ln(1 + \sqrt{2})$ E: NOTA
B: $\frac{1}{2}(\sqrt{2} - \ln(1 + \sqrt{2}))$ D: $\frac{1}{2}$

26. Evaluate

$$\sum_{n=1}^{\infty} \frac{n^2 \cdot 2^n}{n!}$$

- A: $3e^2$ B: $6e^2$ C: $2e^2$ D: e^2 E: NOTA

27. A point is chosen uniformly at random on the interior of the rose curve given by $r = \cos(2\theta)$. Find the expected distance from the origin to this point.

- A: $\frac{16}{9\pi}$ B: $\frac{4}{3\pi}$ C: $\frac{\pi}{8}$ D: $\frac{\pi}{6}$ E: NOTA

28. Evaluate

$$\int_0^{\infty} \frac{1}{(x^2 + 9)^{5/2}} dx$$

- A: $\frac{2}{729}$ B: $\frac{2}{243}$ C: $\frac{1}{729}$ D: $\frac{1}{243}$ E: NOTA

29. How many of the following series converge?

I: $\sum_{n=1}^{\infty} \frac{(-1)^{n^2+n+1}}{\sqrt{n^2+n+1}}$

III: $\sum_{n=1}^{\infty} \frac{\sqrt{n^2+2n+2}-\sqrt{n^2+n+1}}{n\sqrt{n}}$

II: $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

IV: $\sum_{n=1}^{\infty} \frac{(2n)^{n/2}}{n^n}$

- A: 0 B: 1 C: 2 D: 3 E: NOTA

30. Let $f : \left[0, \frac{\pi}{2}\right) \rightarrow [0, \infty)$ be the unique increasing function such that $f(0) = 0$ and for all $0 \leq k < \pi/2$, the arc length of the graph of $f(x)$ between $x = 0$ and $x = k$ is $\ln(\sec(k) + \tan(k))$. Evaluate $\int_0^{\pi/2} f(x) dx$.

- A: $\frac{\pi \ln(2)}{2}$ B: $\frac{\pi \ln(2)}{4}$ C: $\frac{\pi}{2}$ D: $\frac{\pi}{4}$ E: NOTA