

Mu ciphersing answers and solutions
2026 Mu Alpha Theta National Convention

- | | | |
|-------------------|----------------------------|-----------------------------|
| 0. -1 | 6. $\frac{1}{12}$ | 10. $2e^4 - 1$ |
| 1. 200 | | 11. $\ln 2 + \frac{\pi}{8}$ |
| 2. $\ln 2$ | 7. $\frac{8\sqrt{3}}{243}$ | 12. $\frac{1}{2} \ln 6$ |
| 3. $\frac{1}{4}$ | 8. 8 | 13. $\frac{331}{720}$ |
| 4. $1 + \sqrt{2}$ | 9. $\frac{3\pi^2}{4}$ | 14. 68 |
| 5. 1 | | |

$$0. \lim_{x \rightarrow 1} \left(\frac{x^2 - x}{x^2 - 3x + 2} \right) = \lim_{x \rightarrow 1} \left(\frac{x \cancel{(x-1)}}{(x-2)(\cancel{x-1})} \right) = \frac{1}{1-2} = -1$$

1. Since the acceleration is constant 4 and the train began from the stopped position, the velocity function is $4t$, which is positive on the interval $[0, 10]$. Therefore, the distance covered is $\int_0^{10} (4t) dt = 2t^2 \Big|_0^{10} = 200 - 0 = 200$.

2. Using the substitution $u = 1 + \ln x$ and $du = \frac{1}{x} dx$, along with substituting the upper and lower bounds, $\int_1^e \left(\frac{1}{x + x \ln x} \right) dx = \int_1^e \left(\frac{1}{x(1 + \ln x)} \right) dx = \int_1^2 \left(\frac{1}{u} \right) du = \ln |u| \Big|_1^2 = \ln 2 - \ln 1 = \ln 2$.

3. Let x be the length from the base of the ladder to the base of the wall, y be the length from the base of the wall to point of contact of the ladder with the wall, and z be the length of the ladder. Since $x^2 + y^2 = z^2$, $2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$; substituting in the initial values given, along with that the top of the ladder would be 8 ft from the base of the wall, $2(6)(2) + 2(8) \frac{dy}{dt} = 2(10)(1) \Rightarrow \frac{dy}{dt} = -\frac{1}{4}$. Therefore, the top of the ladder is sliding down the wall at a rate of $\frac{1}{4}$ ft/s.

$$4. f(x) = e^{-\left(\frac{(x-1)^2}{4}\right)} \Rightarrow f'(x) = f(x) \cdot \frac{-(x-1)}{2} \Rightarrow f''(x) = f(x) \cdot -\frac{1}{2} + f'(x) \cdot \frac{-(x-1)}{2}$$

$$= f(x) \cdot -\frac{1}{2} + f(x) \cdot \left(\frac{-(x-1)}{2} \right)^2 = f(x) \cdot \left(-\frac{1}{2} + \frac{(x-1)^2}{4} \right). \text{ Since } f(x) \neq 0, f''(x) = 0 \text{ only}$$

when $\frac{1}{2} = \frac{(x-1)^2}{4} \Rightarrow x = 1 \pm \sqrt{2}$, and the sign of $f''(x)$ changes at both of these values,

making them both inflection points. The greatest x -coordinate is then $1 + \sqrt{2}$.

5. These two functions intersect at $(-1, 0)$ and $(1, 0)$, and based on the graphs, which are parabolas that open downward and upward, respectively; therefore, we must have that $-1 \leq y \leq 1$. Further, since the shape is symmetric with respect to the x -axis, we can just find the volume above the x -axis and double that. The length from the right half of $y = 1 - x^2$ to the y -axis is $x = \sqrt{1 - y}$, so the length from the right half of that parabola to the left half is $2\sqrt{1 - y}$, which is the length of the hypotenuse of a cross-section.

Therefore, the area of a cross-section is $\frac{1}{2} \cdot 2\sqrt{1 - y} \cdot \sqrt{1 - y} = 1 - y$, making the volume

$$2 \int_0^1 (1 - y) dy = -(1 - y)^2 \Big|_0^1 = 0 - (-1) = 1.$$

6. Using L'hôpital's Rule, $\lim_{x \rightarrow 0} \left(\frac{c}{1 - \cos x} - \frac{1}{x^2} \right) = \lim_{x \rightarrow 0} \left(\frac{cx^2 - 1 + \cos x}{x^2(1 - \cos x)} \right)$ (which is in $0/0$ form)

$$= \lim_{x \rightarrow 0} \left(\frac{2cx - \sin x}{x^2(\sin x) + 2x(1 - \cos x)} \right) \text{ (still } 0/0) = \lim_{x \rightarrow 0} \left(\frac{2c - \cos x}{x^2(\cos x) + 4x(\sin x) + 2(1 - \cos x)} \right),$$

and since the denominator approaches 0 while the numerator approaches $2c - 1$, in order to still be able to use L'hôpital's Rule, we must have $c = \frac{1}{2}$. Plugging this in and

$$\text{using L'hôpital's Rule, } = \lim_{x \rightarrow 0} \left(\frac{\sin x}{x^2(-\sin x) + 6x(\cos x) + 6(\sin x)} \right) \text{ (still } 0/0)$$

$$= \lim_{x \rightarrow 0} \left(\frac{\cos x}{x^2(-\cos x) + 8x(-\sin x) + 12(\cos x)} \right) = \frac{1}{12}.$$

7. $f(x) = (x - x^3)^3 \Rightarrow f'(x) = 3(x - x^3)^2(1 - 3x^2)$, so the critical numbers in the interval are

$$0, 1, \text{ and } \frac{1}{\sqrt{3}}. \quad f(0) = f(1) = 0, \text{ and } f\left(\frac{1}{\sqrt{3}}\right) = \left(\frac{1}{\sqrt{3}}\right)^3 \left(1 - \frac{1}{3}\right)^3 = \frac{1}{3\sqrt{3}} \cdot \frac{8}{27} = \frac{8\sqrt{3}}{243}.$$

8. $L_a(x) = f(a) + f'(a)(x - a) = a^2 + 2a(x - a) = 2ax - a^2$, so $15 = L_a(4) = 2a(4) - a^2 = 8a - a^2$
 $\Rightarrow 0 = a^2 - 8a + 15 = (a - 3)(a - 5)$, and the sum of the values is $3 + 5 = 8$.

9. Using the tabular method, $\int_0^\pi (x^3 \cos(2x)) dx = \frac{1}{2}x^3 \sin(2x) + \frac{3}{4}x^2 \cos(2x) - \frac{3}{4}x \sin(2x)$

$$-\frac{3}{8}\cos(2x)\Big|_0^\pi = \left(\frac{3\pi^2}{4} - \frac{3}{8}\right) - \left(-\frac{3}{8}\right) = \frac{3\pi^2}{4}.$$

10. $\frac{dy}{dx} = xy + x + y + 1 = (x+1)(y+1) \Rightarrow \frac{dy}{y+1} = (x+1)dx \Rightarrow \ln|y+1| = \frac{1}{2}(x+1)^2 + c$

$$\Rightarrow |y+1| = Ce^{\frac{1}{2}(x+1)^2} \Rightarrow y+1 = \pm Ce^{\frac{1}{2}(x+1)^2} = Ce^{\frac{1}{2}(x+1)^2} \Rightarrow y = Ce^{\frac{1}{2}(x+1)^2} - 1, \text{ so using the}$$

initial condition, $1 = Ce^{\frac{1}{2}(0+1)^2} - 1 = Ce^{\frac{1}{2}} - 1 \Rightarrow C = 2e^{-\frac{1}{2}}$. Therefore,

$$f(2) = 2e^{-\frac{1}{2}}e^{\frac{1}{2}(2+1)^2} - 1 = 2e^4 - 1.$$

11. $\int_0^2 \left(\frac{2x+1}{x^2+4} \right) dx = \int_0^2 \left(\frac{2x}{x^2+4} + \frac{1}{x^2+4} \right) dx = \ln(x^2+4) + \frac{1}{2} \tan^{-1} \frac{x}{2} \Big|_0^2 = \left(\ln 8 + \frac{\pi}{8} \right) - (\ln 4)$
 $= \ln 2 + \frac{\pi}{8}$

12. Using partial fraction decomposition, $\int_{-1}^{-0.5} \left(\frac{-5}{8x^2+14x+3} \right) dx = \int_{-1}^{-0.5} \left(\frac{1}{2x+3} - \frac{2}{4x+1} \right) dx$

$$\frac{1}{2} \ln|2x+3| - \frac{1}{2} \ln|4x+1| \Big|_{-1}^{-0.5} = \left(\frac{1}{2} \ln 2 \right) - \left(-\frac{1}{2} \ln 3 \right) = \frac{1}{2} \ln 6.$$

13. $\int_0^1 (\sin x) dx \approx \int_0^1 \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} \right) dx = \frac{x^2}{2} - \frac{x^4}{4!} + \frac{x^6}{6!} \Big|_0^1 = \left(\frac{1}{2} - \frac{1}{24} + \frac{1}{720} \right) - 0 = \frac{331}{720}$

14. $\sum_{n=1}^{\infty} \left((-1)^{n-1} \cdot \frac{99}{n} \right) = 99 \sum_{n=1}^{\infty} \left(\frac{(-1)^{n-1}}{n} \right)$, which is 99 times a series whose sum is famously

equal to $\ln 2$. Therefore, the sum is $99 \ln 2 = 100 \ln 2 - \ln 2 \approx 100(.693147) - (.693147)$
 $69.3147 - .693147 = 68.621553$, so $a = 68$.