

Where applicable, “E) NOTA” indicates that none of the above answers is correct.

1. A line passing through the points $(k+1, -2k)$ and $(3, -8)$ has slope 2. What is the value of k ?

- A) -2 B) 3 C) $\frac{18}{5}$ D) 4 E) NOTA

2. What is the sum of the solutions to the equation $6x^5 - 19x^3 - 2x^2 + 7x - 11 = 0$?

- A) $-\frac{19}{6}$ B) $-\frac{11}{6}$ C) $\frac{11}{6}$ D) $\frac{19}{6}$ E) NOTA

3. Given the conditions on a five-digit positive integer: (i) greater than 20,000; (ii) the digits are distinct; (iii) the only digits in the integer are 1, 2, 3, 4, or 5; and (iv) the integer is not a multiple of 5; how many such integers exist?

- A) 36 B) 64 C) 78 D) 96 E) NOTA

4. Values b and x are two distinct elements of the set $\{1, 2, 3, 4, 5, 7, 9\}$. How many distinct values exist for $\log_b x$?

- A) 27 B) 31 C) 36 D) 42 E) NOTA

5. How many three-digit numbers have exactly one even digit?

- A) 300 B) 350 C) 360 D) 400 E) NOTA

6. How many subsets of $\{n \in \mathbb{Z} \mid 1 \leq n \leq 2026\}$ exist such that the number of elements in the subset is odd?

- A) 2^{1013} B) 2^{2025} C) 2^{2026} D) $1013!$ E) NOTA

7. What is the greatest positive integer that divides the product of any three consecutive odd positive integers?

- A) 3 B) 5 C) 6 D) 15 E) NOTA

8. The integer 999,999,999,999 is divisible by which of the following?

- A) 49 B) 51 C) 77 D) 81 E) NOTA

9. The vertices of a cube are labeled either 1 or -1 , and each face of the cube is labeled with the product of the numbers that label the vertices of that face. Which of the following cannot be the sum of all fourteen (eight vertices, six faces) labels?

- A) -2 B) 0 C) 2 D) 6 E) NOTA

10. Evaluate: $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$

- A) 0 B) 2 C) 4 D) 8 E) NOTA

11. Which of the following integers is closest to the length of the circumradius of a triangle with sides of lengths 13, 14, and 15?

- A) 8 B) 9 C) 10 D) 11 E) NOTA

12. Two real numbers a and b are selected at random from the interval $[0,10]$. What is the probability that $\frac{b}{a+1}$ is greater than 2?

- A) $\frac{2}{25}$ B) $\frac{4}{25}$ C) $\frac{8}{25}$ D) $\frac{1}{5}$ E) NOTA

13. Evaluate: $\int_1^9 \frac{3}{1+2\sqrt{x}} dx$

- A) $3\ln\frac{2}{3}$ B) $\frac{3}{2}\ln\frac{7}{3}$ C) $4 - \ln 3$ D) $6 + \frac{3}{2}\ln\frac{3}{7}$ E) NOTA

14. If $\int_1^2 (kx^2 - 9x + 1) dx = k$, find the numerical value of k .

- A) $\frac{75}{8}$ B) $-\frac{21}{8}$ C) $-\frac{15}{8}$ D) $\frac{15}{8}$ E) NOTA

15. Simplify, where defined: $\left(\frac{\sec^3 x}{\tan^6 x}\right)^2 \left(\frac{\cos^2 x}{\cot^4 x}\right)^3$

- A) 1 B) $\cot x$ C) $\csc x$ D) $\sec x$ E) NOTA

16. Let f be the function defined by $f\left(\frac{x}{5}\right) = x^2 - 10x + 3$. Find the sum of all values of z for which $f(5z) = 2026$.

- A) $\frac{2}{5}$ B) $\frac{13}{25}$ C) $\frac{3}{5}$ D) $\frac{18}{25}$ E) NOTA

17. Evaluate: $\sum_{n=0}^{\infty} \left(\frac{n}{n^4 + n^2 + 1} \right)$

- A) 0 B) $\frac{1}{6}$ C) $\frac{1}{4}$ D) the series is divergent E) NOTA

18. The product of the complex numbers $3(\cos 123^\circ + i \sin 123^\circ)$ and $4(\cos 282^\circ + i \sin 282^\circ)$ can be written in the form $a + bi$, where a and b are real. Find the value of $|ab|$.

- A) 12 B) 24 C) 48 D) 72 E) NOTA

19. The remainder when $P(x) = \left(\frac{x^3 - 14}{13}\right)^{2025}$ is divided by $x^2 - 4x + 3$ can be written in the form $a + bx$, where a and b are real. Find the value of $|b - a|$.

- A) 1 B) 2 C) 3 D) 4 E) NOTA

20. The polynomial $p(x)$ with real coefficients has degree 4, and $p(n) = \frac{1}{n+1}$ for $n = 1, 2, 3, 4, 5$. Compute the value of $p(6)$.

- A) $\frac{1}{7}$ B) $\frac{1}{6}$ C) $\frac{1}{3}$ D) $\frac{6}{7}$ E) NOTA

21. Let the function f be defined on the non-negative integers by

$$f(n) = \begin{cases} \lfloor n^{1/2} \rfloor, & \text{if } n \equiv 0 \text{ or } 1 \pmod{3} \\ \lfloor n^{3/2} \rfloor + 1, & \text{if } n \equiv 2 \pmod{3} \end{cases}, \text{ where } \lfloor x \rfloor \text{ equals the greatest integer } n \text{ such that}$$

$n \leq x$. Find the smallest positive integer k such that $f^k(2026) = 1$, where $f^k(n)$ represents $\underbrace{(f \circ f \circ \dots \circ f)}_{k \text{ copies of } f}(n)$.

- A) 3 B) 5 C) 7 D) 9 E) NOTA

22. Given that $\log 3 = A$, $\log 5 = B$, and $\log 11 = C$, which of the following is equivalent to $\log 1.\overline{36}$?

- A) $A - B - C$ B) $A - B + C$ C) $B - A + C$ D) $B - A - C$ E) NOTA

23. A circle is tangent to the coordinate axes and also contains the point $(8,1)$. Which of the following could be the length of the radius of this circle?

- A) 7 B) 9 C) 12 D) 13 E) NOTA

24. Let z be the complex number $z = \frac{1}{2}(\sqrt{2+\sqrt{2}} + i\sqrt{2-\sqrt{2}})$. If $w = z^{2026} + z^{14} + 1$, then $|w|$ lies in which of the following intervals?

- A) $[1.4, 1.5]$ B) $[1.7, 1.8]$ C) $[2, 2.1]$ D) $[2.2, 2.3]$ E) NOTA

25. A triangular region in the coordinate plane is bounded by the x -axis, the y -axis, and line L with negative slope that passes through the point $(2,5)$. What is the minimum area of this triangular region?

- A) 5 B) 10 C) 15 D) 20 E) NOTA

26. Let $ABCD$ be a trapezoid with bases $\overline{BC}$ and $\overline{AD}$ of lengths 25 and 49, respectively, and suppose that the ratio of the length of leg $\overline{AB}$ to that of leg $\overline{CD}$ is 5:7. Find the length of $\overline{AC}$.

- A) 32 B) 35 C) 36 D) 40 E) NOTA

27. Find the minimum value of $\tan^2 \theta + 4\sec \theta + 2026$.

- A) 2022 B) 2023 C) 2024 D) 2025 E) NOTA

28. A particle is located at the point $(1,2,3)$ at time $t=0$. The particle travels in a straight line in the direction of the vector $\langle 3,-1,1 \rangle$, and at time $t=0$, the speed of the particle is 2. The particle also has constant acceleration $\langle 3,-1,1 \rangle$. If the position of the particle at time $t=2\sqrt{11}$ can be written as (a,b,c) , where a , b , and c are real, find the value of $a+b+c$.

- A) $35 + \frac{5\sqrt{11}}{2}$ B) $39 + 3\sqrt{11}$ C) 84 D) 90 E) NOTA

29. Three spheres, all of different sizes, are tangent to the same plane at points M , A , and T in the plane. The three spheres are also tangent to each other. If triangle MAT has sides of lengths 18, 21, and 24, find the product of the lengths of the radii of the three spheres.

- A) 1134 B) 2248 C) 2646 D) 5292 E) NOTA

30. Suppose that M is a 3×3 matrix whose entries are chosen randomly from the set $\{0,1\}$. What is the probability that matrix M has an inverse?

- A) $\frac{51}{256}$ B) $\frac{31}{128}$ C) $\frac{21}{64}$ D) $\frac{11}{32}$ E) NOTA