

Where applicable, “E) NOTA” indicates that none of the above answers is correct.

Assume the domain and range of all functions are limited to the real numbers, unless specified.

1. Evaluate: $\lim_{x \rightarrow 0} \left(\frac{\sqrt{1+4x} - \sqrt{1-2x}}{x} \right)$

- A) 0 B) 1 C) 2 D) 3 E) NOTA

2. Which of the following is equal to $\frac{d}{dx} \left(\frac{\sin^2 x}{1 + \cot x} + \frac{\cos^2 x}{1 + \tan x} \right)$?

- A) $-\cos(2x)$ B) $-\sec(2x)$ C) $-\sin(2x)$ D) $-\csc(2x)$ E) NOTA

3. For which value of k does the equation $e^{2x} = k\sqrt{x}$ have exactly one solution?

- A) $2\sqrt{e}$ B) e C) e^2 D) 1 E) NOTA

4. Let P and Q be quadratic polynomials with real coefficients satisfying $P(Q(x)) = P(x) \cdot Q(x)$ for all real numbers x . If $Q(2025) = 1$, find the value of $Q(2026)$.

- A) 0 B) 1 C) 2024 D) 2025 E) NOTA

5. Air is being pumped into a spherical balloon so that its volume increases at a rate of 100 cubic inches per minute. How fast is the radius of the balloon increasing, in inches per minute, when the diameter has length 50 inches?

- A) $\frac{1}{50\pi}$ B) $\frac{1}{25\pi}$ C) $\frac{1}{10\pi}$ D) $\frac{1}{5\pi}$ E) NOTA

6. Suppose that function f is twice differentiable. If $f(2) = 10$ and $f'(x) = x^2 \cdot f(x)$ for all real numbers x , find $f''(2)$.

- A) 20 B) 100 C) 200 D) 400 E) NOTA

7. Let x and y be real numbers such that $4x^2 + 9y^2 = 36$. If the maximum and minimum values of $x^2 + xy + y^2$ are M and m , respectively, find $M - m$.

- A) $2\sqrt{61}$ B) $\sqrt{61}$ C) 13 D) $\frac{13}{2}$ E) NOTA

8. Find the y -intercept of the tangent line to the function $y = f(x) = \sqrt{4x+1}$ at $x = 2$.

- A) $\frac{2}{3}$ B) 3 C) 5 D) $\frac{5}{3}$ E) NOTA

9. Find the smallest positive integer m such that $m^{\frac{1}{m}} \geq n^{\frac{1}{n}}$ for all positive integers $n \geq m$.

- A) 2 B) 3 C) 5 D) 6 E) NOTA

10. Suppose that a sequence $\{a_n\}$ of real numbers satisfies $a_n = a_{2n+1}$ and $a_{2n} = a_{2n-1} + a_{2n+1}$ for all positive integers n . If $a_{111} = 2025$, what is the value of a_1 ?

- A) 1 B) 45 C) 135 D) 405 E) NOTA

11. Let $I_n = \int_0^{\frac{\pi}{4}} (\tan^n x) dx$. Which of the following statements is true for $n \geq 2$?

- A) $I_n + I_{n-1} = \frac{1}{n-1}$ B) $I_n + I_{n-2} = \frac{1}{n-1}$ C) $I_n - I_{n-2} = \frac{n}{n-1}$ D) $I_n - I_{n-1} = \frac{n}{n-1}$

E) NOTA

12. Consider function f defined by $f(x) = \begin{cases} x^2, & \text{if } x \geq 0 \\ -x^2, & \text{if } x < 0 \end{cases}$. Which of the following statements are true?

I. f is continuous at $x = 0$.

II. f is differentiable at $x = 0$.

III. f' is continuous at $x = 0$.

IV. f' is differentiable at $x = 0$.

- A) I only B) I and II only C) I, II, and III only D) I, II, III, and IV E) NOTA

13. Evaluate: $\int_0^{\ln 2} \frac{1}{1+e^x} dx$

- A) $\ln 2$ B) $\ln \frac{3}{2}$ C) $\ln \frac{4}{3}$ D) $\ln \frac{5}{4}$ E) NOTA

14. If $f(x) = \int_1^x te^{2t-x} dt$, find the value of $f'(1)$.

- A) 1 B) e C) e^2 D) e^3 E) NOTA

15. If $f(x) = \cos(x^3)$, find the value of $f^{(6)}(0)$.

- A) 90 B) 180 C) -180 D) -360 E) NOTA

16. Find the sum of the series: $\sum_{n=2}^{\infty} \ln\left(1 - \frac{1}{n^2}\right)$

- A) $-\ln 2$ B) $\ln 2$ C) 1 D) -1 E) NOTA

17. Evaluate: $\int_0^{\pi} (\sqrt{1+\sin x} + \sqrt{1-\sin x}) dx$

- A) $2\sqrt{3}$ B) $4\sqrt{3}$ C) $2\sqrt{2}$ D) $4\sqrt{2}$ E) NOTA

18. Find the area of the region between the curves $y = \sin x$ and $y = \cos x$ on the interval $\left[0, \frac{\pi}{2}\right]$.

- A) $2\sqrt{3}$ B) $2\sqrt{2} - 2$ C) $3\sqrt{2}$ D) 2 E) NOTA

19. Evaluate: $\int_0^{\infty} \frac{x^2}{(1+x^2)^2} dx$

- A) $\frac{\pi}{4}$ B) $\frac{\pi}{2}$ C) $\frac{3\pi}{4}$ D) $\frac{3\pi}{2}$ E) NOTA

20. Let P be a polynomial of degree 9 with real coefficients such that for integers k from 1 to 10, inclusive, $P(k) = \frac{(k+1)^2}{k}$. Find the value of $12 \cdot P(12)$.

- A) 122 B) 145 C) 159 D) 172 E) NOTA

21. For a cubic function $f(x) = ax^3 + bx^2 + cx + d$, where a, b, c , and d are real numbers, what condition on the coefficients guarantees two distinct local extrema?

- A) $b^2 > 3ac$ B) $b^2 < 3ac$ C) $b^2 > 4ac$ D) $b^2 < 4ac$ E) NOTA

22. Let F_n be the Fibonacci sequence generated by $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$, with $F_0 = 0$ and $F_1 = 1$. If $g(x) = \sum_{n=1}^{\infty} F_n x^n$, which of the following is equivalent to $g(x)$?

- A) $\frac{1}{1+x^2}$ B) $\frac{x}{1+x+x^2}$ C) $\frac{x}{1-x-x^2}$ D) $\frac{1}{1-x+x^2}$ E) NOTA

23. Find the radius of convergence for the series $\sum_{n=1}^{\infty} \left(\frac{n!}{n^n} x^n \right)$.

- A) $\frac{1}{e}$ B) 1 C) $\sqrt{e}$ D) e E) NOTA

24. Evaluate: $\sum_{n=1}^{\infty} \left(\frac{1}{2^n} \sin \frac{n\pi}{3} \right)$

- A) $\frac{1}{4}$ B) $\frac{\sqrt{2}}{2}$ C) $\frac{\sqrt{3}}{3}$ D) 1 E) NOTA

25. If f is a function defined as $f(x) + 2f\left(\frac{1}{x}\right) = 3x$ for all real numbers $x \neq 0$, find the value of $f(3)$.

- A) $\frac{3}{5}$ B) $-\frac{7}{6}$ C) $-\frac{14}{3}$ D) $-\frac{7}{3}$ E) NOTA

26. Let $f: \mathbb{N} \rightarrow \mathbb{N}$ be a function such that $f(f(f(n))) = n + 3$. Find the sum of all possible values of $f(100)$.

- A) 412 B) 418 C) 424 D) 432 E) NOTA

27. Let $R(x)$ be the remainder when $x^6 + x^4 + x^2 + 1$ is divided by $(x-1)^3$. Find the value of $R(0)$.

- A) 22 B) 14 C) 44 D) 32 E) NOTA

28. Find the minimum distance between the parabola with equation $y^2 = 2x$ and the point $(1, 4)$.

- A) 1 B) 2 C) $\sqrt{5}$ D) $\sqrt{6}$ E) NOTA

29. The curve with equation $x^3 + y^3 - 3xy = 0$ has a horizontal tangent line at point (a, b) , where $a > 0$. Find the value of the product ab .

- A) 1 B) 2 C) 3 D) 4 E) NOTA

30. If $f_n(x) = |x - n|$ for any nonnegative integer n , evaluate $\int_{-6}^6 ((f_3 \circ f_2 \circ f_1 \circ f_0)(x)) dx$.

- A) 16 B) 20 C) 24 D) 28 E) NOTA