

All uppercase letter variables are positive integers unless otherwise stated. All fractions containing uppercase letter variables are in lowest terms. NOTA means “None of the Above.”

~~~~~ Good luck, and have fun! ~~~~~

1. Which of the following graphs is different from the others, up to translations and rotations?
  - A. A unit circle in the Cartesian plane
  - B.  $z = 1$  in the Argand plane
  - C.  $r = 1$  in the polar coordinate system
  - D. A great circle of the unit ball
  - E. NOTA
  
2. The point  $P = (a, b)$  is referring to a point with polar coordinates or a point in the Cartesian plane. This location is the same in both systems. Of the choices, which of the following is the most restrictive true statement that can be made about  $P$ ?
  - A.  $P$  exists.
  - B.  $P$  is the origin.
  - C.  $P$  is on the  $x$ -axis in the Cartesian plane.
  - D.  $P$  is on the  $y$ -axis in the Cartesian plane.
  - E. NOTA
  
3. The graph of  $x = \cos(2t)$ ,  $y = \sin^2(t)$  is a portion of which of the following?
  - A. Line
  - B. Hyperbola
  - C. Parabola
  - D. Ellipse
  - E. NOTA
  
4. The graph of  $x = \log_2 t$ ,  $y = \log_t 2$  exists only in which quadrant(s)?
  - A. I
  - B. I and III
  - C. II
  - D. II and IV
  - E. NOTA
  
5. Find the area enclosed by a regular hexagon inscribed in the unit circle.
  - A.  $\sqrt{3}$
  - B.  $\frac{4\sqrt{3}}{3}$
  - C.  $\frac{3\sqrt{3}}{2}$
  - D.  $2\sqrt{3}$
  - E. NOTA

6. Find the area enclosed by the largest square that can be inscribed in the graph of  $y = \pm \frac{2}{x}$ .
- A. 4                                      C. 16                                      E. NOTA  
B. 8                                      D. 32
7. Find the slope of the tangent line to  $r = e^{2\theta}$  at  $\theta = \frac{\pi}{4}$ .
- A. -3                                      C. -1                                      E. NOTA  
B. 1                                      D. 3
8. Find the area of the region bounded by the graphs of  $r = e^{2\theta}$  and  $\theta = \frac{\pi}{2}$  for  $\theta$  in the interval  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ .
- A.  $\frac{1}{8}$                                       C.  $\frac{e^{2\pi} - e^{-2\pi}}{8}$                                       E. NOTA  
B.  $\frac{e^{2\pi} - 1}{8}$                                       D.  $\frac{e^{2\pi}}{8}$
9. Find the length of the curve given by  $x = 3t - 2$ ,  $y = 4t - 3$  in the fourth quadrant.
- A.  $\frac{1}{5}$                                       C. 1                                      E. NOTA  
B.  $\frac{5}{12}$                                       D. 5
10. The solutions to  $4x^2 + bx + c = 0$  can be parametrized as  $\sin(\theta)$  and  $\cos(\theta)$  for some  $\theta \in \left[0, \frac{\pi}{2}\right]$ . Find the value of  $8(b^2 - 8c)$ .
- A. 32                                      C. 128                                      E. NOTA  
B. 64                                      D. 256
11. I have a 14-foot tall painting of Lisa from Genshin Impact in my house, depicting her casting a spell; it's called Lisa's Mana. The bottom of the painting is 18 feet off the ground. If I stand 24 feet back from the wall on which the painting is hanging, find the tangent of my viewing angle (from the ground, from bottom to top of the painting) of Lisa's Mana.
- A.  $\frac{7}{12}$                                       C.  $\frac{5}{12}$                                       E. NOTA  
B.  $\frac{3}{4}$                                       D.  $\frac{7}{24}$

12. Two real numbers are independently uniformly randomly selected from the range  $[0,1]$ . Find the probability that the positive difference between the numbers exceeds  $\frac{1}{2}$ .

A.  $\frac{1}{8}$   
B.  $\frac{1}{4}$

C.  $\frac{1}{3}$   
D.  $\frac{1}{2}$

E. NOTA

13. Of the choices, which of the following is the most restricting true statement that can be made about the number  $\frac{e^{i\theta}+1}{e^{i\theta}-1}$ , assuming  $e^{i\theta} \neq 1$ ?

- A. It is on the unit circle.  
B. It is a real number.  
C. It is a pure imaginary number.  
D. It is on the complex coordinate axes.  
E. NOTA

14. Which of the following is equal to  $\operatorname{arcsinh}(x)$ ?

*Hint:*  $\sinh(i\theta) = i\sin(\theta)$ .

A.  $\ln(\sqrt{x^2+1}-x)$   
B.  $\ln(x-\sqrt{x^2-1})$

C.  $\ln(x+\sqrt{x^2-1})$   
D.  $\ln(\sqrt{x^2+1}+x)$

E. NOTA

15. Find  $\frac{d^2y}{dx^2}$  if  $x = 2026\sin(2026\theta)$  and  $y = 2026\cos(2026\theta)$ .

A.  $-\frac{2026^2}{y^3}$   
B.  $-\frac{2026^2}{x^3}$

C.  $-\frac{2026}{y^3}$   
D.  $-\frac{2026}{x^3}$

E. NOTA

16. Find  $\frac{dy}{dx}$  at  $t = \frac{\pi}{4}$  if  $x = e^{2t}\cos(t)$  and  $y = e^{4t}\sin^2(t)$ .

A.  $3\sqrt{2}e^{\pi/2}$   
B.  $4\sqrt{2}e^{\pi/2}$

C.  $7\sqrt{2}e^{\pi/2}$   
D.  $8\sqrt{2}e^{\pi/2}$

E. NOTA

17. Triangle  $ABC$  has vertices  $A = (0,0)$ ,  $B = (14,0)$ , and  $C = (x,y)$ . Suppose  $\frac{dx}{dt} = 3$  and  $\frac{dy}{dt} = 4$ . Find  $\frac{d[ABC]}{dt}$  when  $x = 9$  and  $y = 12$ .
- A. 21                                      C. 49                                      E. NOTA  
B. 28                                      D. 84
18. Triangle  $ABC$  has side lengths 13, 14, and 15. Point  $L$  is chosen such that the sum of the squares of the distances from  $L$  to the sides of the triangle is minimized. Which of the following values is closest to this sum?
- A. 44                                      C. 48                                      E. 52  
B. 46                                      D. 50
19. Find the length of the curve given by  $r = \csc(\theta)$  for  $\frac{\pi}{6} \leq \theta \leq \frac{\pi}{3}$ .
- A.  $\frac{\sqrt{3}}{6}$                                       C.  $\frac{\sqrt{3}}{2}$                                       E. NOTA  
B.  $\frac{\sqrt{3}}{3}$                                       D.  $\frac{2\sqrt{3}}{3}$
20. Determine the area of the region in quadrant I bounded by the graphs of  $2r^4 = \csc(2\theta) \sec^2(\theta)$ ,  $\theta = 0$ , and  $\theta = \frac{\pi}{4}$ .
- A.  $\frac{1}{2}$                                       C. 1                                      E. NOTA  
B.  $\frac{1}{\sqrt{2}}$                                       D.  $\sqrt{2}$
21. Cal Raleigh hits a baseball from a height of 30 meters off the flat ground (he's very tall), with an initial velocity of 50 meters per second and a launch angle of  $30^\circ$  above the horizontal. If acceleration due to gravity is 10 meters per second squared, find the horizontal distance (in meters) the ball will have traveled when it first touches the ground.
- A.  $100\sqrt{3}$                                       C.  $150\sqrt{3}$                                       E. NOTA  
B.  $125\sqrt{3}$                                       D. 300

22. The curvature of a curve  $y = f(x)$  at the point  $(x, f(x))$  is given by the formula

$\kappa(x) = \frac{|y''|}{(1+(y')^2)^{3/2}}$ , where  $y'$  and  $y''$  are the first and second derivatives of  $y$  with respect to  $x$ . The maximum curvature of  $y = e^{2x}$  occurs at  $x = c$ . Find  $c$ .

- A.  $-\frac{3}{4}\ln(2)$                       C.  $-\frac{1}{4}\ln(2)$                       E. NOTA  
B.  $-\frac{1}{2}\ln(2)$                       D.  $-\frac{1}{8}\ln(2)$

23. Two vertices of a triangle are  $(-1,0)$  and  $(1,0)$ . The third vertex is on the curve  $y = \sqrt{1-x^2}$ . Among all such triangles, find the average value of the enclosed area.

- A.  $\frac{\pi}{4}$                                       C.  $\frac{\pi}{6}$                                       E. NOTA  
B.  $\frac{\pi}{5}$                                       D.  $\frac{\pi}{7}$

24. If  $\int_5^{17} \frac{x}{\sqrt{2x^2-8x-1}} dx = A + \sqrt{2}\ln(\sqrt{B} + C)$ , where  $A$ ,  $B$ , and  $C$  are all positive integers, find the value of  $A + B + C$ .

- A. 20                                      C. 32                                      E. NOTA  
B. 26                                      D. 36

25. Let the total area of the region(s) inside both the graphs of  $r^2 = 8\cos(2\theta)$  and  $r = 4\sin(\theta)$  in the polar plane equal  $\frac{A\pi}{B} - C\sqrt{D} + E$ , for positive integers  $A$ ,  $B$ ,  $C$ ,  $D$ , and  $E$  in simplest fractional and radical forms. Find the product  $ABCDE$ .

- A. 288                                      C. 432                                      E. NOTA  
B. 384                                      D. 576

26. If the region contained within the loop of the graph of  $x^3 + y^2 = 3x^2$  has area  $\frac{A\sqrt{B}}{C}$ , for positive integers  $A$ ,  $B$ , and  $C$  in simplest fractional and radical forms, find  $A + B + C$ .

- A. 8                                          C. 32                                      E. NOTA  
B. 20                                      D. 56

27. Evaluate:  $\tan\left(\int_0^{\frac{\pi}{2}} \frac{dx}{\sin(x)+2\cos(x)+3}\right)$ .

*Hint:* Use  $t = \tan\left(\frac{x}{2}\right)$ .

A.  $\frac{1}{4}$

B.  $\frac{1}{3}$

C.  $\frac{1}{2}$

D.  $\frac{2}{3}$

E. NOTA

28. Find  $\frac{1}{I}$  if  $I = \int_0^1 \frac{t^5(1-t^2)^3}{(1+t^2)^9} dt$ .

A. 768

B. 1024

C. 1536

D. 3072

E. NOTA

29. Find the product of the lengths of the diagonals of a regular hexagon with side length 1.

A. 27

B.  $24\sqrt{3}$

C. 72

D. 216

E. NOTA

30. Regular octagon *HEPTAGON* has sides of length 8. Segments *ET* and *PA* intersect at *X*. Let the area of heptagon *HEXAGON* equal  $Y + Z\sqrt{2}$  for positive integers *Y* and *Z*. Find the value of  $Y - Z$ .

A. -32

B. -16

C. 16

D. 32

E. NOTA