

BCABC CDCBC DBCDA ABCDA CAAAD CBCDE

- B:** $z = 1$ in the Argand plane represents a single number and is thus a single point. All other choices are circles of radius 1.
- C:** $(a, b) = (a \cos(b), b \sin(b))$. Looking at the x -coordinate, $a = a \cos(b)$, so $a = 0$ or $b = 0$. The first of these cases is uniquely the origin, so $b = 0$. The second of these cases is true for any value of a , so $P = (x, 0)$ and must be on the x -axis.
- A:** $\cos(2t) = 1 - 2\sin^2(t)$, so $x = 1 - 2y$, which is a portion of a line because of the restricted ranges of cosine and sine.
- B:** $y = \frac{1}{x}$. Since $\log_2 t$ has a range of all real numbers, it is the full graph.
- C:** The radius of a hexagon is equal to its side length. $\frac{3 \cdot 1^2 \sqrt{3}}{2} = \frac{3\sqrt{3}}{2}$.
- C:** The square is rotated 45° and has side length equal to the distance between the two pieces of $y = \frac{2}{x}$. This is between $(\sqrt{2}, \sqrt{2})$ and $(-\sqrt{2}, -\sqrt{2})$, which is 4. The area of the square is 16.
- D:** $r = e^{\pi/2}$, $r' = 2e^{\pi/2}$, $\sin(\theta) = \cos(\theta) = \frac{1}{\sqrt{2}} \cdot \frac{r \cos(\theta) + r' \sin(\theta)}{-r \sin(\theta) + r' \cos(\theta)} = \frac{3e^{\pi/2}}{e^{\pi/2}} = 3$.
- C:** The upper bound of the region is clearly $\theta = \frac{\pi}{2}$, and the lower bound is also on the vertical axis, at $\theta = -\frac{\pi}{2}$. $\frac{1}{2} \int_{-\pi/2}^{\pi/2} (e^{2\theta})^2 d\theta = \frac{1}{2} \int_{-\pi/2}^{\pi/2} e^{4\theta} d\theta = \left[\frac{e^{4\theta}}{4} \right]_{-\pi/2}^{\pi/2} = \frac{e^{2\pi} - e^{-2\pi}}{8}$.
- B:** $y = \frac{4}{3}(x + 2) - 3 = \frac{4x}{3} - \frac{1}{3}$. The intercepts of this are $(\frac{1}{4}, 0)$ and $(0, -\frac{1}{3})$. The distance between these points is $\sqrt{\frac{1}{16} + \frac{1}{9}} = \frac{5}{12}$.
- C:** By Vieta's, $\sin(\theta) + \cos(\theta) = -\frac{b}{4}$ and $\sin(\theta)\cos(\theta) = \frac{c}{4}$. $1 = \sin^2(\theta) + \cos^2(\theta) = (\sin(\theta) + \cos(\theta))^2 - 2\sin(\theta)\cos(\theta) = \frac{b^2}{16} - \frac{c}{2}$. Thus, $b^2 - 8c = 16$. $8 \cdot 16 = 128$.
- D:** $\theta = \arctan\left(\frac{4}{3}\right) - \operatorname{arccot}\left(\frac{3}{4}\right)$, so $\tan \theta = \frac{\frac{4}{3} - \frac{3}{4}}{1 + \left(\frac{4}{3}\right)\left(\frac{3}{4}\right)} = \frac{7}{24}$.
- B:** In the unit square, the relevant parts of the region are above the line $y = x + \frac{1}{2}$ and below the line $y = x - \frac{1}{2}$. These are two isosceles right triangles with leg length $\frac{1}{2}$, for a total area of $\frac{1}{4}$.
- C:** Let $e^{i\theta} = a + bi$ where $a^2 + b^2 = 1$. The expression is $\frac{a+1+bi}{a-1+bi} = \frac{(a+1+bi)(a-1-bi)}{(a-1)^2+b^2} = \frac{a^2+b^2-2ib-1}{a^2+b^2-2a+1} = -\frac{2ib}{2-2a} = \frac{ib}{a-1}$. Note that it is explicitly given that $a \neq 1$. This a pure imaginary number.

14. **D:** Using the complex definition of sine, $i\sin(\theta) = \frac{e^{i\theta} - e^{-i\theta}}{2}$, so $x = \sinh(\theta) = \frac{e^\theta - e^{-\theta}}{2}$. Solving for θ , $e^{2\theta} - 1 - 2xe^\theta = 0$, so $e^\theta = \frac{2x + \sqrt{4x^2 + 4}}{2} = x + \sqrt{x^2 + 1}$; note the conjugate is rejected since it is negative. Thus, $\theta = \ln(x + \sqrt{x^2 + 1})$.
15. **A:** $x^2 + y^2 = 2026^2$. Deriving implicitly, $2x + 2y \cdot \frac{dy}{dx} = 0$ so $\frac{dy}{dx} = -\frac{x}{y}$. Deriving again, $2 + 2\left(\frac{dy}{dx}\right)^2 + 2y\left(\frac{d^2y}{dx^2}\right) = 0$, so $\frac{d^2y}{dx^2} = \frac{1}{2y}\left(-2 - \frac{2x^2}{y^2}\right) = \frac{-y^2 - x^2}{y^3} = -\frac{2026^2}{y^3}$.
16. **A:** Notice that $x^2 = e^{4t}\cos^2(t) = e^{4t} - e^{4t}\sin^2(t) = e^{4t} - y$. Deriving $x^2 + y - e^{4t} = 0$ with respect to x , $2x + \frac{dy}{dx} - 4e^{4t} \cdot \frac{dt}{dx} = 0$, so $\frac{dy}{dx} = 4e^{4t} \div \frac{dx}{dt} - 2x$. When $t = \frac{\pi}{4}$, this is $4e^\pi \div \frac{dx}{dt} - \sqrt{2}e^{\pi/2} \cdot \frac{dx}{dt} = e^{2t}(2\cos(t) - \sin(t))$, which at $t = \frac{\pi}{4}$ evaluates to $\frac{1}{\sqrt{2}}e^{\pi/2}$. Combining, $4\sqrt{2}e^{\pi/2} - \sqrt{2}e^{\pi/2} = 3\sqrt{2}e^{\pi/2}$.
17. **B:** The area of the triangle is $7y$, since the base is horizontal. The rate of change of the area is $7\frac{dy}{dt} = 28$.
18. **C:** Let the distances from L to the sides with length a , b , and c be d_a , d_b , and d_c . Let the area of the triangle be Δ . Drawing segments from L to the vertices of the triangle, $2\Delta = ad_a + bd_b + cd_c$. By Cauchy-Schwarz, $(a^2 + b^2 + c^2)(d_a^2 + d_b^2 + d_c^2) \geq (ad_a + bd_b + cd_c)^2$, so $d_a^2 + d_b^2 + d_c^2 \geq \frac{4\Delta^2}{a^2 + b^2 + c^2}$. Plugging in values, $\frac{4 \cdot 84^2}{13^2 + 14^2 + 15^2} \approx \frac{4 \cdot 84^2}{3 \cdot 14^2} = \frac{4}{3} \cdot 36 = 48$, which is very close to 47.837 ...
19. **D:** $r\sin(\theta) = 1$, so $y = 1$. $\theta = \frac{\pi}{6}$ and $\theta = \frac{\pi}{3}$ convert to the lines $y = \frac{x}{\sqrt{3}}$ and $y = x\sqrt{3}$, which intersect the curve at $x = \sqrt{3}$ and $x = \frac{1}{\sqrt{3}}$. The difference between these is $\frac{2\sqrt{3}}{3}$.
20. **A:** $4r^4\sin^3(\theta)\cos(\theta) = 1$, so $4x^3y = 1$ and this is the graph of $y = \frac{1}{4x^3}$. $\theta = \frac{\pi}{4}$ is the line $y = x$, which intersects the graph at $x = \frac{1}{\sqrt{2}}$. Part of the area is an isosceles right triangle with leg length $\frac{1}{\sqrt{2}}$, which has area $\frac{1}{4}$. The rest of the area is $\int_{1/\sqrt{2}}^{\infty} \frac{dx}{4x^3} = \frac{1}{4}$, so the total area is $\frac{1}{2}$.
21. **C:** Vertically, $a = -10$, $v = -10t + 25$, and $y = -5t^2 + 25t + 30$. This will land when $y = 0$, so $t^2 - 5t - 6 = 0$ and $t = 6$. Horizontal velocity remains constant throughout, so $25\sqrt{3} \cdot 6 = 150\sqrt{3}$ meters. Nice homer, Big Dumper!
22. **A:** We wish to maximize $\kappa(x) = \frac{4e^{2x}}{(1+4e^{4x})^{3/2}}$. Taking advantage of a bijection, let $u = e^x$, so $\kappa(u) = \frac{4u^2}{(1+4u^4)^{3/2}}$. Deriving, $\kappa'(u) = \frac{8u - 64u^5}{(1+4u^4)^{5/2}}$. This is 0 when $8u - 64u^5 = 0$, or $u = 2^{-3/4}$ (ignoring extraneous solutions). This corresponds to $c = -\frac{3}{4}\ln(2)$.

23. **A:** The height of the triangle is $\sqrt{1-x^2}$. The average area is $\frac{1}{2} \int_{-1}^1 \sqrt{1-x^2} dx = \frac{\pi}{4}$.
24. **A:** $\int_5^{17} \frac{x}{\sqrt{2(x-2)^2-9}} dx = \int_3^{15} \frac{u+2}{\sqrt{2u^2-9}} du$. Let $u = \frac{3\sec(\theta)}{\sqrt{2}}$ so $du = \frac{3\sec(\theta)\tan(\theta)}{\sqrt{2}} d\theta$. The integral becomes $\frac{1}{\sqrt{2}} \int_{\pi/4}^{\operatorname{arcsec}(5\sqrt{2})} \frac{3\sec(\theta)+2\sqrt{2}}{3\tan(\theta)} \cdot \frac{3\sec(\theta)\tan(\theta)}{\sqrt{2}} = \frac{1}{2} \int_{\pi/4}^{\operatorname{arcsec}(5\sqrt{2})} (3\sec^2(\theta) + 2\sqrt{2}\sec(\theta)) d\theta = \left[\frac{3}{2}\tan(\theta) + \sqrt{2}\ln(\sec(\theta) + \tan(\theta)) \right]_{\pi/4}^{\operatorname{arcsec}(5\sqrt{2})}$. Note that $\operatorname{arcsec}(5\sqrt{2}) = \arctan(7)$. This evaluates to $\left(\frac{3}{2} \cdot 7 + \sqrt{2}\ln(5\sqrt{2} + 7) \right) - \left(\frac{3}{2} \cdot 1 + \sqrt{2}\ln(\sqrt{2} + 1) \right) = \frac{3}{2} \cdot 6 + \sqrt{2}\ln(2\sqrt{2} + 3) = 9 + 2\ln(\sqrt{8} + 3)$. $9 + 8 + 3 = 20$.
25. **D:** In the first quadrant (representing half the area), the graphs intersect when $8\cos(2\theta) = 16\sin^2(\theta)$. Solving $8 - 16\sin^2(\theta) = 16\sin^2(\theta)$ gives $\sin(\theta) = \frac{1}{2}$, or $\theta = \frac{\pi}{6}$. The integral can be split into two parts. $\int_0^{\pi/6} 8\sin^2(\theta) d\theta = \int_0^{\pi/6} (4 - 4\cos(2\theta)) d\theta = \frac{2\pi}{3} - [2\sin(2\theta)]_0^{\pi/6} = \frac{2\pi}{3} - \sqrt{3}$. $\int_{\pi/6}^{\pi/4} 4\cos(2\theta) d\theta = [2\sin(2\theta)]_{\pi/6}^{\pi/4} = 2 - \sqrt{3}$. Adding the pieces together gives a total area in the first quadrant of $\frac{2\pi}{3} - 2\sqrt{3} + 2$, and doubling this for the whole graph gives $\frac{4\pi}{3} - 4\sqrt{3} + 4$. $4 \cdot 3 \cdot 4 \cdot 3 \cdot 4 = 576$.
26. **C:** Convert to polar. $r^3\cos^3(\theta) + r^2\sin^2(\theta) = 4r^2\cos^2(\theta)$, so $r = 4\sec(\theta) - \tan^2(\theta)\sec(\theta) = \sec(\theta)(3 - \tan^2(\theta))$. This equals 0 at $\theta = \pm \frac{\pi}{3}$. $\frac{1}{2} \int_{-\pi/3}^{\pi/3} \sec^2(\theta)(3 - \tan^2(\theta))^2 d\theta = \int_0^{\pi/3} \sec^2(\theta)(3 - \tan^2(\theta))^2 d\theta = \int_0^{\sqrt{3}} (3 - u^2)^2 du = \int_0^{\sqrt{3}} (u^4 - 6u^2 + 9) du = \left[\frac{u^5}{5} - 2u^3 + 9u \right]_0^{\sqrt{3}} = \frac{9\sqrt{3}}{5} - 6\sqrt{3} + 9\sqrt{3} = \frac{24\sqrt{3}}{5}$. $24 + 3 + 5 = 32$.
27. **B:** The given Weierstrass substitution can be used to find $\sin(x) = \frac{2t}{1+t^2}$, $\cos(x) = \frac{1-t^2}{1+t^2}$, and $dx = \frac{2}{1+t^2} dt$. The integral becomes $\int_0^1 \frac{2}{2t+2(1-t^2)+3(1+t^2)} dt = 2 \int_0^1 \frac{dt}{t^2+2t+5} = 2 \int_1^2 \frac{du}{u^2+4} = \left[\arctan \frac{u}{2} \right]_1^2 = \frac{\pi}{4} - \arctan\left(\frac{1}{2}\right)$. The tangent of this is $\frac{1-1/2}{1+1 \cdot 1/2} = \frac{1}{3}$.
28. **C:** Note that $64I = \int_0^1 \frac{32t^5}{(1+t^2)^5} \cdot \frac{(1-t^2)^3}{(1+t^2)^3} \cdot \frac{2}{1+t^2} dt$. A reverse-Weierstrass substitution can be used, where the integral becomes $\frac{1}{64} \int_0^{\pi/2} \sin^5(\theta) \cdot \cos^3(\theta) d\theta = \frac{1}{64} \int_0^{\pi/2} \sin^5(\theta)(1 - \sin^2(\theta))\cos(\theta) d\theta = \frac{1}{64} \int_0^1 (u^5 - u^7) du = \frac{1}{64} \left(\frac{1}{6} - \frac{1}{8} \right) = \frac{1}{64} \cdot \frac{1}{24} = \frac{1}{1536}$. $\frac{1}{1536} \cdot \frac{1}{I} = 1536$.

29. **D:** Of the $\frac{6 \cdot 3}{2} = 9$ diagonals of a regular hexagon, 3 of them span two radii between opposite vertices and have length 2, and the other 6 of them connect a vertex to another vertex two sides over and have length $\sqrt{3}$. $2^3 \cdot \sqrt{3}^6 = 216$.
30. **E:** The area of the octagon is $2 \cdot 64(1 + \sqrt{2}) = 128 + 128\sqrt{2}$. The areas of triangles EPT and APT need to be subtracted; those are both $\frac{1}{2} \cdot 8 \cdot 4\sqrt{2} = 16\sqrt{2}$. However, this subtraction over-counts triangle PTX . Set P as the origin. Then $A = (8 + 4\sqrt{2}, 4\sqrt{2})$, so PA is on the line $y = x(\sqrt{2} - 1)$. X has x -coordinate 4, so its y -coordinate is $4\sqrt{2} - 4$, and the area of triangle PTX is $16\sqrt{2} - 16$. $(128 + 128\sqrt{2}) - 2 \cdot 16\sqrt{2} + (16\sqrt{2} - 16) = 112 + 112\sqrt{2}$. $112 - 112 = 0$.