

	A)	B)	C)	D)
0)	0	1	1	0
1)	17	13	11	17
2)	1	1	1	1
3)	$\frac{8}{9}$	$(-2, -4)$	above	$\frac{307}{72}$
4)	8	$\sqrt{10}$	$\frac{64}{27}$	$\frac{15\sqrt[3]{10}}{8}$
5)	$1+2\sqrt{3}$	$\frac{5}{2}$	$\frac{3-\sqrt{3}}{3}$	-2
6)	$\frac{1}{3}$	$e-2$	$\ln 2$	$\frac{4-\pi}{2}$
7)	$\frac{4}{3}$	$\frac{40}{3}$	$\ln \sqrt{2}$	$\ln \frac{81}{128}$
8)	2	$\frac{9\pi}{2}$	6π	3
9)	$\frac{81\pi}{2}$	$\frac{972\pi}{5}$	64π	$\frac{1088\pi}{15}$
10)	$y = 3e^x$	$y = 4e^{\frac{1}{2}x^2} - 1$	$y = 3\sqrt{x^2 + 1}$	$y = \ln(e^x + e^3 - 1)$
11)	no	yes	yes	yes
12)	-3	0	-4	$\frac{4}{3}$
13)	4	$(-\log_2(\ln 2), \infty)$	$-\frac{5\pi}{24}$	$\frac{11\pi}{30}$
14)	$\frac{3}{2}$	$\frac{4}{3}$	$\frac{11}{24}$	$\frac{5}{12}$

0) For A), by L'Hôpital's Rule and continuity of the sine function, $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = \lim_{x \rightarrow 0} \frac{\sin x}{1} = \sin 0 = 0$. For B), by L'Hôpital's Rule and continuity of the cosine function, $\lim_{x \rightarrow 0} \frac{\sin x}{x} = \lim_{x \rightarrow 0} \frac{\cos x}{1} = \cos 0 = 1$. For C), by L'Hôpital's Rule and continuity of the exponential function, $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = \lim_{x \rightarrow 0} \frac{e^x}{1} = e^0 = 1$. For D), by continuity of polynomials, $\lim_{x \rightarrow 0} x = 0$.

1) Since polynomials are continuous, each of these limits is just the evaluation of the function at the value x is approaching. Therefore, for A), $\lim_{x \rightarrow 0} (x^3 - 2x^2 - 3x + 17) = 0^3 - 2 \cdot 0^2 - 3 \cdot 0 + 17 = 17$; for B), $\lim_{x \rightarrow 1} (x^3 - 2x^2 - 3x + 17) = 1^3 - 2 \cdot 1^2 - 3 \cdot 1 + 17 = 13$; for C), $\lim_{x \rightarrow 2} (x^3 - 2x^2 - 3x + 17) = 2^3 - 2 \cdot 2^2 - 3 \cdot 2 + 17 = 11$; for D), $\lim_{x \rightarrow 3} (x^3 - 2x^2 - 3x + 17) = 3^3 - 2 \cdot 3^2 - 3 \cdot 3 + 17 = 17$.

2) All four of these expressions represent the value of the derivative of $y = x^3 - 2x^2 - 3x + 17$ when $x = 2$. Therefore, for A), B), C), and D), since the derivative is $y' = 3x^2 - 4x - 3$, the answer is $3 \cdot 2^2 - 4 \cdot 2 - 3 = 1$.

3) First, $f(x) = \frac{x^2}{x+1} \Rightarrow f'(x) = \frac{(x+1)(2x) - x^2(1)}{(x+1)^2} = \frac{x^2 + 2x}{(x+1)^2} = 1 - \frac{1}{(x+1)^2}$ and $f'(x) = 1 - (x+1)^{-2} \Rightarrow f''(x) = 2(x+1)^{-3} = \frac{2}{(x+1)^3}$. Therefore, for A), $f'(2) = 1 - \frac{1}{(2+1)^2} = 1 - \frac{1}{9} = \frac{8}{9}$.

For B), using sign analysis on f' , f is increasing on $(-\infty, -2)$, is decreasing on $(-2, -1)$, has a vertical asymptote at $x = -1$, is decreasing on $(-1, 0)$, and is increasing on $(0, \infty)$. Therefore,

the local maximum is at $x = -2$. Since $f(-2) = \frac{(-2)^2}{-2+1} = -4$, the local maximum is at the

ordered pair $(-2, -4)$. For C), since $f''(x) = \frac{2}{(x+1)^3}$, for $x < -1$, $f''(x) < 0$, meaning f is

concave down on this interval. Therefore, the tangent line for x -values in the given interval lies

above the function. For D), $f(5) = \frac{5^2}{5+1} = \frac{25}{6}$ and $f'(5) = 1 - \frac{1}{(5+1)^2} = 1 - \frac{1}{36} = \frac{35}{36}$, so the

tangent line to $y = f(x)$ at the point where $x = 5$ is $y - \frac{25}{6} = \frac{35}{36}(x - 5) \Rightarrow y = \frac{35}{36}x - \frac{25}{36}$.

Therefore, $f(5.1) \approx \frac{35}{36}(5.1) - \frac{25}{36} = \frac{153.5}{36} = \frac{307}{72}$.

4) For A), since the semicircle satisfies $x^2 + y^2 = 36$, the distance between $(0, -2)$ and an arbitrary point (x, y) on the semicircle is $d = \sqrt{(x-0)^2 + (y-(-2))^2} = \sqrt{x^2 + y^2 + 4y + 4}$

$=\sqrt{36+4y+4}=\sqrt{4y+40}$, but this is an increasing expression for the semicircle's y -values, $0 \leq y \leq 6$; therefore, the maximum distance is $\sqrt{4 \cdot 6 + 40} = \sqrt{64} = 8$. For B), the distance between $(4,0)$ and an arbitrary point (x,y) on the function is $d = \sqrt{(x-4)^2 + (y-0)^2}$
 $= \sqrt{x^2 - 8x + 16 + (x^3)^2} = \sqrt{x^2 - 8x + 16 + x^6}$. Since this radicand, $x^2 - 8x + 16 + x^6$, is only non-negative, we can simply maximize the radicand: $d^2 = x^2 - 8x + 16 + x^6 \Rightarrow (d^2)' = 2x - 8 + 6x^5$
 $\Rightarrow (d^2)' = 0$ if $x = 1$ (the other solutions to this are imaginary). Due to the nature of the graph of $y = x^3$, this point must minimize the distance, meaning the minimum distance is $d = \sqrt{1^2 - 8 \cdot 1 + 16 + 1^6} = \sqrt{10}$. For C), letting x equal the length of a base edge and y equal the height of the prism, we must have $8x + 4y = 16 \Rightarrow y = 4 - 2x$. Therefore, the volume of the rectangular prism is $V = x^2y = x^2(4 - 2x) = 4x^2 - 2x^3$, and the domain of V is $0 < x < 2$, so we only need to look in this interval. Since $V' = 8x - 6x^2 = 2x(4 - 3x)$, $V' = 0 \Rightarrow x = \frac{4}{3} \Rightarrow y = \frac{4}{3}$, and $V' > 0$ for $0 < x < \frac{4}{3}$ while $V' < 0$ for $\frac{4}{3} < x < 2$, so this is where the volume is a maximum, which is $V = \left(\frac{4}{3}\right)^3 = \frac{64}{27}$. For D), letting x and y be the lengths parallel to their corresponding axes, the area is $A = xy = x(5 - x^3) = 5x - x^4$ with domain $0 < x < \sqrt[3]{5}$, so $A' = 5 - 4x^3 \Rightarrow A' = 0$ if $x^3 = \frac{5}{4} \Rightarrow x = \frac{\sqrt[3]{10}}{2}$, and $A' > 0$ for $0 < x < \frac{\sqrt[3]{10}}{2}$ while $A' < 0$ for $\frac{\sqrt[3]{10}}{2} < x < \sqrt[3]{5}$, so this is where the maximum area occurs, which is $A = \frac{\sqrt[3]{10}}{2} \left(5 - \frac{5}{4}\right) = \frac{\sqrt[3]{10}}{2} \cdot \frac{15}{4} = \frac{15\sqrt[3]{10}}{8}$.

5) For A), since f is continuous everywhere with $f(4) = -3 < 0$ and $f(5) = 4 > 0$, Intermediate Value Theorem guarantees a $c \in (4,5)$ such that $f(c) = 0$ (not that you need to know this for this problem, you just need to solve for the root in that interval). Therefore, $0 = x^2 - 2x - 11 \Rightarrow 12 = x^2 - 2x + 1 = (x-1)^2 \Rightarrow \pm 2\sqrt{3} = x - 1 \Rightarrow 1 \pm 2\sqrt{3} = x$, but only $1 + 2\sqrt{3}$ is in the given interval, making that the value of c . For B), since f is differentiable everywhere, Mean Value Theorem guarantees a $c \in (2,3)$ such that $f'(c) = \frac{f(3) - f(2)}{3 - 2} \Rightarrow 4c - 3 = \frac{10 - 3}{1} = 7 \Rightarrow 4c = 10$

$\Rightarrow c = \frac{5}{2}$. For C), since f is differentiable everywhere with $f(0) = -8 = f(1)$, Rolle's Theorem

guarantees a $c \in (0, 1)$ such that $f'(c) = 0 \Rightarrow 0 = 3c^2 - 6c + 2 = 3(c-1)^2 - 1 \Rightarrow (c-1)^2 = \frac{1}{3}$

$\Rightarrow c-1 = \pm \frac{\sqrt{3}}{3} \Rightarrow c = 1 \pm \frac{\sqrt{3}}{3} = \frac{3 \pm \sqrt{3}}{3}$, but only $\frac{3-\sqrt{3}}{3}$ is in the given interval, making that the

value of c . For D), this value is guaranteed since f is continuous on closed, bounded interval $[-5, -1]$. Since $f'(x) = 3x^2 - 6x - 24 = 3(x-4)(x+2)$, the only critical number of f in the given interval is -2 , meaning the maximum value of f on this interval occurs either here or at one of the endpoints of the interval. Since $f(-5) = -68$, $f(-1) = 32$, and $f(-2) = 40$, the value of c where f achieves its maximum in this interval is $c = -2$.

6) For A), $\int_0^1 (x^2 - 6x + 3) dx = \left(\frac{1}{3}x^3 - 3x^2 + 3x \right) \Big|_0^1 = \left(\frac{1}{3} - 3 + 3 \right) - (0 - 0 + 0) = \frac{1}{3}$. For B),

$\int_0^1 (e^x - 2x) dx = (e^x - x^2) \Big|_0^1 = (e - 1) - (1 - 0) = e - 2$. For C), using a simple substitution,

$\int_0^1 \left(\frac{2x}{x^2 + 1} \right) dx = (\ln|x^2 + 1|) \Big|_0^1 = (\ln 2) - (\ln 1) = \ln 2$. For D), first perform the division of the

function: $\frac{2x^2}{x^2 + 1} = 2 - \frac{2}{x^2 + 1}$. Then $\int_0^1 \left(\frac{2x^2}{x^2 + 1} \right) dx = \int_0^1 \left(2 - \frac{2}{x^2 + 1} \right) dx = (2x - 2 \tan^{-1} x) \Big|_0^1$

$= \left(2 - 2 \left(\frac{\pi}{4} \right) \right) - (0 - 0) = 2 - \frac{\pi}{2} = \frac{4 - \pi}{2}$.

7) It is easiest to evaluate these limits by turning them into integrals. For A),

$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\left(\left(\frac{2i}{n} \right)^3 - \left(\frac{2i}{n} \right)^2 + \left(\frac{2i}{n} \right) - 1 \right) \left(\frac{2}{n} \right) \right) = \int_0^2 (x^3 - x^2 + x - 1) dx = \left(\frac{1}{4}x^4 - \frac{1}{3}x^3 + \frac{1}{2}x^2 - x \right) \Big|_0^2$

$= \left(4 - \frac{8}{3} + 2 - 2 \right) - (0 - 0 + 0 - 0) = \frac{4}{3}$. For B), $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\left(\left(1 + \frac{2i}{n} \right)^3 - \left(1 + \frac{2i}{n} \right)^2 + \left(1 + \frac{2i}{n} \right) - 1 \right) \left(\frac{2}{n} \right) \right)$

$= \int_1^3 (x^3 - x^2 + x - 1) dx = \left(\frac{1}{4}x^4 - \frac{1}{3}x^3 + \frac{1}{2}x^2 - x \right) \Big|_1^3 = \left(\frac{81}{4} - 9 + \frac{9}{2} - 3 \right) - \left(\frac{1}{4} - \frac{1}{3} + \frac{1}{2} - 1 \right) = \frac{40}{3}$. For

C), $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\left(\tan \left(\frac{\pi i}{4n} \right) \right) \left(\frac{\pi}{4n} \right) \right) = \int_0^{\frac{\pi}{4}} (\tan x) dx = (\ln|\sec x|) \Big|_0^{\frac{\pi}{4}} = \ln \sqrt{2} - \ln 1 = \ln \sqrt{2}$. For D),

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\left(\frac{6 - \frac{i}{n}}{\left(\frac{i}{n}\right)^3 - 9\left(\frac{i}{n}\right)^2 + 26\left(\frac{i}{n}\right) - 24} \right) \left(\frac{1}{n}\right) \right) = \int_0^1 \left(\frac{6-x}{x^3 - 9x^2 + 26x - 24} \right) dx, \text{ but we must use}$$

partial fraction decomposition in order to evaluate this integral, so factoring the denominator,

$$\text{then using Heaviside Method, } \frac{6-x}{x^3 - 9x^2 + 26x - 24} = \frac{6-x}{(x-2)(x-3)(x-4)} = \frac{2}{x-2} - \frac{3}{x-3} + \frac{1}{x-4}.$$

$$\text{Therefore, } \int_0^1 \left(\frac{6-x}{x^3 - 9x^2 + 26x - 24} \right) dx = \int_0^1 \left(\frac{2}{x-2} - \frac{3}{x-3} + \frac{1}{x-4} \right) dx = (2\ln|x-2| - 3\ln|x-3| + \ln|x-4|) \Big|_0^1 \\ = (2\ln 1 - 3\ln 2 + \ln 3) - (2\ln 2 - 3\ln 3 + \ln 4) = 4\ln 3 - 7\ln 2 = \ln 81 - \ln 128 = \ln \frac{81}{128}.$$

8) Each of these can be written as an integral that can either be worked using the Fundamental Theorem of Calculus or, in a couple of them, much more easily by interpreting in terms of area. For A), this is the right triangular region with vertices at the origin, $(2,0)$, and $(0,2)$, so the area

is $\frac{1}{2} \cdot 2 \cdot 2 = 2$. For B), this is the region bounded by the semicircle below the x-axis of the circle centered at the origin with radius 3 along with the x-axis, so this is half of such a disk—the area, then, is $\frac{1}{2} \cdot \pi \cdot 3^2 = \frac{9\pi}{2}$ (notice that the integral of this function would give a negative number,

but the question asks for the area, which cannot be negative). For C), $y = \frac{\sqrt{63 - 54x - 9x^2}}{4}$

$$\Rightarrow 4y = \sqrt{63 - 54x - 9x^2} \Rightarrow 16y^2 = 63 - 54x - 9x^2 \Rightarrow 16y^2 + 9(x^2 + 6x + 9) = 63 + 81 = 144$$

$$\Rightarrow 16y^2 + 9(x+3)^2 = 144 \Rightarrow \frac{y^2}{9} + \frac{(x+3)^2}{16} = 1, \text{ so this integral is the area of the upper half of the}$$

region bounded by this ellipse, making the value $\frac{1}{2} \cdot \pi \cdot 3 \cdot 4 = 6\pi$. For D), this is a trapezoidal

region with altitude $5 - 3 = 2$. The base lengths are 1 and 2 (x-values corresponding to the intersections of the two horizontal lines with the slanted line), so the area is $\frac{1}{2} \cdot 2 \cdot (1 + 2) = 3$.

9) All of these can be found using integrals as well. For A), using the disk method, the volume is

$$\pi \int_0^9 (\sqrt{x})^2 dx = \pi \int_0^9 x dx = \pi \left(\frac{1}{2} x^2 \right) \Big|_0^9 = \pi \left(\frac{81}{2} - 0 \right) = \frac{81\pi}{2}.$$

For B), using the shells method, the volume is $2\pi \int_0^9 (x \cdot \sqrt{x}) dx = 2\pi \int_0^9 x^{\frac{3}{2}} dx = 2\pi \left(\frac{2}{5} x^{\frac{5}{2}} \right) \Big|_0^9 = 2\pi \left(\frac{486}{5} - 0 \right) = \frac{972\pi}{5}.$ For $x = y^2 - 6y + 10$

$= (y-3)^2 + 1 \Rightarrow y = 3 \pm \sqrt{x-1}$ (+ gives the upper half, - gives the lower half), and the vertex of this parabola is at the point (1,3). Therefore, for C), using the washers method, the volume is

$$\pi \int_1^5 \left((3 + \sqrt{x-1})^2 - (3 - \sqrt{x-1})^2 \right) dx = \pi \int_1^5 12\sqrt{x-1} dx = 12\pi \left(\frac{2}{3} (x-1)^{\frac{3}{2}} \right) \Big|_1^5 = 8\pi(8-0) = 64\pi.$$

For D), using the shells method, along with the substitution $u = \sqrt{x-1}$, the volume is

$$\begin{aligned} 2\pi \int_1^5 \left(x \cdot \left((3 + \sqrt{x-1}) - (3 - \sqrt{x-1}) \right) \right) dx &= 2\pi \int_1^5 (x \cdot 2\sqrt{x-1}) dx = 4\pi \int_0^2 ((u^2 + 1) \cdot u \cdot 2u) du \\ &= 8\pi \int_0^2 (u^4 + u^2) du = 8\pi \left(\frac{1}{5} u^5 + \frac{1}{3} u^3 \right) \Big|_0^2 = 8\pi \left(\left(\frac{32}{5} + \frac{8}{3} \right) - (0+0) \right) = \frac{1088\pi}{15}. \end{aligned}$$

10) Each of these integrals is separable. For A), $\int \frac{dy}{y} = \int dx \Rightarrow \ln|y| = x + c \Rightarrow |y| = Ce^x \Rightarrow y = Ce^x$,

and with the initial condition, $3 = Ce^0 = C \Rightarrow y = 3e^x$. For B), $\frac{dy}{dx} = x(y+1) \Rightarrow \int \frac{dy}{y+1} = \int x dx$

$\Rightarrow \ln|y+1| = \frac{1}{2} x^2 + c \Rightarrow |y+1| = Ce^{\frac{1}{2} x^2} \Rightarrow y+1 = Ce^{\frac{1}{2} x^2} \Rightarrow y = Ce^{\frac{1}{2} x^2} - 1$, and with the initial

condition, $3 = Ce^{\frac{1}{2} 0^2} - 1 = C - 1 \Rightarrow C = 4 \Rightarrow y = 4e^{\frac{1}{2} x^2} - 1$. For C), $\int \frac{dy}{y} = \int \frac{x}{x^2+1} dx \Rightarrow \ln|y|$

$= \frac{1}{2} \ln|x^2+1| + c = \frac{1}{2} \ln(x^2+1) + c = \ln\sqrt{x^2+1} + c \Rightarrow |y| = C\sqrt{x^2+1} \Rightarrow y = C\sqrt{x^2+1}$, and with the

initial condition, $3 = C\sqrt{0^2+1} = C \Rightarrow y = 3\sqrt{x^2+1}$. For D), $\int e^y dy = \int e^x dx \Rightarrow e^y = e^x + c$

$\Rightarrow y = \ln(e^x + c)$, and with the initial condition, $3 = \ln(e^0 + c) = \ln(1+c) \Rightarrow e^3 = 1+c \Rightarrow e^3 - 1 = c$

$\Rightarrow y = \ln(e^x + e^3 - 1)$.

11) For A), $\int_1^{\infty} \frac{x}{x^2+1} dx = \lim_{t \rightarrow \infty} \int_1^t \frac{x}{x^2+1} dx = \lim_{t \rightarrow \infty} \left(\ln(x^2+1) \right) \Big|_1^t = \lim_{t \rightarrow \infty} \left(\ln(t^2+1) - (\ln 2) \right) = \infty$, so this

series diverges by the Integral Test (and the answer is thus “no”). For B), $\lim_{n \rightarrow \infty} \left| \frac{\frac{(n+1)!}{(n+1)^{n+1}}}{\frac{n!}{n^n}} \right|$

$$= \lim_{n \rightarrow \infty} \left(\frac{(n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{n!} \right) = \lim_{n \rightarrow \infty} \left(\frac{n^n}{(n+1)^n} \right) = \lim_{n \rightarrow \infty} \left(\frac{1}{\left(\frac{n+1}{n} \right)^n} \right) = \lim_{n \rightarrow \infty} \left(\frac{1}{\left(1 + \frac{1}{n} \right)^n} \right) = \frac{1}{e} < 1, \text{ so this series}$$

converges (absolutely) by the Ratio Test (and the answer is thus “yes”). For C), the radicands of the numerator and denominator are both linear with slope 2, so they are increasing at the same rate. However, the numerator is a cube root while the denominator is a square root, so the denominator is increasing more quickly than the numerator as $n \rightarrow \infty$; therefore, $\frac{\sqrt[3]{2n+1}}{\sqrt{2n-1}}$

is decreasing as $n \rightarrow \infty$. Further, $= \lim_{n \rightarrow \infty} \left(\frac{\sqrt[3]{2n+1}}{\sqrt{2n-1}} \right) = \lim_{n \rightarrow \infty} \left(\frac{\sqrt[6]{(2n+1)^2}}{\sqrt[6]{(2n-1)^3}} \right)$

$$= \sqrt[6]{\lim_{n \rightarrow \infty} \left(\frac{4n^2 + 4n + 1}{8n^3 - 12n^2 + 6n - 1} \right)} = \sqrt[6]{0} = 0, \text{ so this series converges by the Alternating Series Test}$$

(and the answer is thus “yes”). For D), $0 \leq \left| \frac{\cos n}{1+n^2} \right| \leq \frac{1}{1+n^2} < \frac{1}{n^2}$, and $\sum_{n=1}^{\infty} \left(\frac{1}{n^2} \right)$ converges by p -

Series test, so $\sum_{n=1}^{\infty} \left(\left| \frac{\cos n}{1+n^2} \right| \right)$ converges by Direct Comparison Test. Therefore, $\sum_{n=1}^{\infty} \left(\frac{\cos n}{1+n^2} \right)$

converges absolutely (and the answer is thus “yes”).

12) For A), $f(x, y) = 2x^2y - e^{xy} + \cos x \Rightarrow f_x(x, y) = 4xy - ye^{xy} - \sin x \Rightarrow f_x(0, 3) = 4 \cdot 0 \cdot 3 - 3e^{0.3}$

$-\sin 0 = 0 - 3 - 0 = -3$. For B), $f(x, y) = 2x^2y - e^{xy} + \cos x \Rightarrow f_y(x, y) = 2x^2 - xe^{xy} + 0 \Rightarrow f_y(0, 3)$

$= 2 \cdot 0^2 - 0e^{0.3} + 0 = 0 - 0 + 0 = 0$. For C), $g(x, y) = \frac{y}{x^2+1} - \sin^{-1}(xy) + \ln\left(\frac{y}{x+1}\right) = y(x^2+1)^{-1}$

$$\begin{aligned}
 & -\sin^{-1}(xy) + \ln y - \ln(x+1) \Rightarrow g_x(x, y) = -y(x^2+1)^{-2}(2x) - \frac{1}{\sqrt{1-(xy)^2}}(y) + 0 - \frac{1}{x+1} = -\frac{2xy}{(x^2+1)^2} \\
 & -\frac{y}{\sqrt{1-(xy)^2}} - \frac{1}{x+1} \Rightarrow g_x(0, 3) = -\frac{2 \cdot 0 \cdot 3}{(0^2+1)^2} - \frac{3}{\sqrt{1-(0 \cdot 3)^2}} - \frac{1}{0+1} = 0 - 3 - 1 = -4. \text{ For D),} \\
 & g(x, y) = \frac{y}{x^2+1} - \sin^{-1}(xy) + \ln\left(\frac{y}{x+1}\right) = \frac{y}{x^2+1} - \sin^{-1}(xy) + \ln y - \ln(x+1) \Rightarrow g_y(x, y) = \frac{1}{x^2+1} \\
 & -\frac{1}{\sqrt{1-(xy)^2}}(x) + \frac{1}{y} - 0 = \frac{1}{x^2+1} - \frac{x}{\sqrt{1-(xy)^2}} + \frac{1}{y} \Rightarrow g_y(0, 3) = \frac{1}{0^2+1} - \frac{0}{\sqrt{1-(0 \cdot 3)^2}} + \frac{1}{3} = 1 - 0 \\
 & + \frac{1}{3} = \frac{4}{3}.
 \end{aligned}$$

$$\begin{aligned}
 13) \text{ For A), } f(x) &= 3x^2 + e^x + \tan x + \ln(x+1) + \tan^{-1} x \Rightarrow f'(x) = 6x + e^x + \sec^2 x + \frac{1}{x+1} + \frac{1}{x^2+1} \\
 \Rightarrow f'(0) &= 6 \cdot 0 + e^0 + \sec^2 0 + \frac{1}{0+1} + \frac{1}{0^2+1} = 0 + 1 + 1 + 1 + 1 = 4. \text{ For B), } g(x) = 2^x - x \Rightarrow g'(x) \\
 &= 2^x \ln 2 - 1 \Rightarrow g'(x) = 0 \text{ when } 2^x \ln 2 - 1 = 0 \Rightarrow 2^x = \frac{1}{\ln 2} = (\ln 2)^{-1} \Rightarrow x = \log_2(\ln 2)^{-1} = -\log_2(\ln 2).
 \end{aligned}$$

Further, if $x < -\log_2(\ln 2)$, $g'(x) < 0$, while if $x > -\log_2(\ln 2)$, $g'(x) > 0$, so g is increasing on

$$\begin{aligned}
 & (-\log_2(\ln 2), \infty). \text{ For C), } \int_{-1}^1 \left(\frac{1}{(x+1)^2+4} - \frac{1}{\sqrt{4-x^2}} \right) dx = \left(\frac{1}{2} \tan^{-1} \left(\frac{x+1}{2} \right) - \sin^{-1} \frac{x}{2} \right) \Big|_{-1}^1 = \left(\frac{1}{2} \tan^{-1} 1 \right. \\
 & \left. - \sin^{-1} \frac{1}{2} \right) - \left(\frac{1}{2} \tan^{-1} 0 - \sin^{-1} \left(-\frac{1}{2} \right) \right) = \left(\frac{1}{2} \cdot \frac{\pi}{4} - \frac{\pi}{6} \right) - \left(0 - \left(-\frac{\pi}{6} \right) \right) = \frac{\pi}{8} - \frac{\pi}{6} - \frac{\pi}{6} = -\frac{5\pi}{24}. \text{ For D), the}
 \end{aligned}$$

intersections of the two curves are the points $(0,0)$ and $(1,1)$, so using the shells method, for x -values between 0 and 1, the radius is $1-x$ and the height is $\sqrt{x}-x$, so the volume is

$$\begin{aligned}
 2\pi \int_0^1 ((1-x)(\sqrt{x}-x^2)) dx &= 2\pi \int_0^1 \left(x^{\frac{1}{2}} - x^2 - x^{\frac{3}{2}} + x^3 \right) dx = 2\pi \left(\frac{2}{3} x^{\frac{3}{2}} - \frac{1}{3} x^3 - \frac{2}{5} x^{\frac{5}{2}} + \frac{1}{4} x^4 \right) \Big|_0^1 = 2\pi \left(\left(\frac{2}{3} \right. \right. \\
 & \left. \left. - \frac{1}{3} - \frac{2}{5} + \frac{1}{4} \right) - (0-0-0+0) \right) = 2\pi \left(\frac{11}{60} \right) = \frac{11\pi}{30}. \text{ (Note: this could also be worked using the}
 \end{aligned}$$

washers method on an integral in terms of y)

$$14) \text{ For A), the first two terms of the Maclaurin series of } e^x \text{ are } 1+x, \text{ so } \int_0^1 e^x dx \approx \int_0^1 (1+x) dx$$

$= \left(x + \frac{1}{2}x^2 \right) \Big|_0^1 = \left(1 + \frac{1}{2} \right) - (0 + 0) = \frac{3}{2}$. For B), the first two terms of the Maclaurin series of e^{x^2}

are $1 + x^2$, so $\int_0^1 e^{x^2} dx \approx \int_0^1 (1 + x^2) dx = \left(x + \frac{1}{3}x^3 \right) \Big|_0^1 = \left(1 + \frac{1}{3} \right) - (0 + 0) = \frac{4}{3}$. For C), the first two

terms of the Maclaurin series of $\sin x$ are $x - \frac{x^3}{3!} = x - \frac{1}{6}x^3$, so $\int_0^1 (\sin x) dx \approx \int_0^1 \left(x - \frac{1}{6}x^3 \right) dx$

$= \left(\frac{1}{2}x^2 - \frac{1}{24}x^4 \right) \Big|_0^1 = \left(\frac{1}{2} - \frac{1}{24} \right) - (0 + 0) = \frac{11}{24}$. For D), the first two terms of the Maclaurin series

of $\tan^{-1} x$ are $x - \frac{x^3}{3} = x - \frac{1}{3}x^3$, so $\int_0^1 (\tan^{-1} x) dx \approx \int_0^1 \left(x - \frac{1}{3}x^3 \right) dx = \left(\frac{1}{2}x^2 - \frac{1}{12}x^4 \right) \Big|_0^1 = \left(\frac{1}{2} - \frac{1}{12} \right)$

$-(0 + 0) = \frac{5}{12}$.