

Where applicable, “E) NOTA” indicates that none of the above answers is correct.

Unless otherwise specified, a “die” or “dice” are fair and six-sided, numbered 1 through 6; a “coin” or “coins” are fair with a heads on one side and a tails on the other, and the result(s) of a “coin flip” or “coin flips” is/are either heads or tails (with equal probability) on each flip; and a “deck of cards” is a standard, 52-card deck with a deuce through ace each of clubs, diamonds, hearts, and spades—an ace may be counted as a high or low card in a poker hand, but not simultaneously both. Additionally, for playing cards, “rank” refers to the value of the card (2, 3, ..., K, A).

1. What is the probability that you get a prime number when rolling a die?

- A) $\frac{1}{3}$ B) $\frac{1}{2}$ C) $\frac{2}{3}$ D) $\frac{5}{6}$ E) NOTA

2. In how many orders can one distinctly arrange the letters in the word COMBINATORICS?

- A) $\frac{13!}{8}$ B) $\frac{13!}{4}$ C) $\frac{13!}{2}$ D) $13!$ E) NOTA

3. Let S be the set $\{f, r, i, s, c, o\}$. How many nonempty subsets of S are there?

- A) 63 B) 64 C) 719 D) 720 E) NOTA

4. Three turquoise marbles, four burgundy marbles, and five mauve marbles are put in a bag. One marble is selected from the bag at random. What is the probability that the marble selected is turquoise?

- A) $\frac{1}{12}$ B) $\frac{1}{4}$ C) $\frac{1}{3}$ D) $\frac{2}{3}$ E) NOTA

5. Seven students from the same school are at a math tournament. Only four students may participate in the team round. In how many ways can the four students participating in the team round be chosen?

- A) 16 B) 35 C) 210 D) 840 E) NOT

6. An urn contains 15 marbles: 10 maroon and 5 indigo. Three marbles are selected from the urn at random without replacement. What is the probability that all three are maroon?

- A) $\frac{8}{27}$ B) $\frac{3}{13}$ C) $\frac{24}{91}$ D) $\frac{19}{27}$ E) NOTA

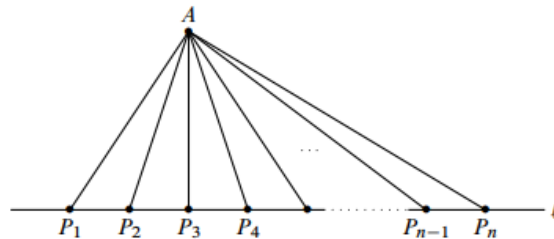
7. An urn contains 40 scarlet marbles, 27 emerald marbles, 26 cerulean marbles, and 7 violet marbles. What is the smallest number of marbles that must be drawn at random to guarantee that at least 15 marbles drawn have the same color?

- A) 39 B) 43 C) 50 D) 86 E) NOTA

8. In a survey of 100 people, 60 indicate that they get their news from watching television, and 70 indicate that they get their news from the internet. Of those that get new from the internet, 70% also watch television. Compute the number of people surveyed who do not get their news from either television or internet.

- A) 17 B) 19 C) 21 D) 23 E) NOTA

9. Let A be point not on line ℓ . On line ℓ , mark points $P_1, P_2, P_3, \dots, P_n$ as shown in the figure to the right. Construct $\overline{AP_k}$ for $k = 1, 2, \dots, n$. The figure then contains 136 total triangles of all sizes. Compute the value of n .



- A) 16 B) 18 C) 45 D) 135 E) NOTA

10. A gardener wants to plant some colorful flowers in a row. They have four goldenrods, four lilacs, and three roses. We can assume each goldenrod is indistinguishable from the other goldenrods, and likewise for the lilacs and roses. How many distinguishable arrangements of the flowers are there?

- A) 240 B) 630 C) 9240 D) 11550 E) NOTA

11. A gardener wants to plant some colorful flowers in a row. They have four goldenrods, four lilacs, and three roses. We can assume each goldenrod is indistinguishable from the other goldenrods, and likewise for the lilacs and roses. How many distinguishable arrangements of the flowers have roses on each end of the row?

- A) 240 B) 630 C) 9240 D) 11550 E) NOTA

12. Nine students are at a math tournament. A teacher is to select 4 students from 5 seniors and 4 juniors to form a four-student team for the tournament's team round. If the selection is made randomly, what is the probability that the team includes at least one senior and at least one junior?

- A) $\frac{20}{21}$ B) $\frac{23}{24}$ C) $\frac{119}{120}$ D) 1 E) NOTA

13. How many terms are in the expanded and simplified form of $(w + x + y + z)^{10}$?

- A) 120 B) 210 C) 240 D) 1048576 E) NOTA

14. You flip a coin until you get heads. What is the expected number of flips you will make?

- A) 1 B) 1.5 C) 2 D) 2.5 E) NOTA

15. A restaurant manager is creating the schedule for a catering event. The manager needs 8 of 13 servers to work the event, but two servers, Jayden and Riley, refuse to work with one another. In how many ways can the manager assign servers to work the event?

- A) 396 B) 429 C) 792 D) 825 E) NOTA

16. A gardener wants to plant some colorful flowers in a row. They have four goldenrods, four lilacs, and three roses. We can assume each goldenrod is indistinguishable from the other goldenrods, and likewise for the lilacs and roses. How many distinguishable arrangements of the flowers have all the lilacs next to each other?

- A) 240 B) 630 C) 9240 D) 11550 E) NOTA

17. An ant is sitting at each corner of a regular heptagon. Each ant then walks along exactly one side of the heptagon, with each having an equal probability of choosing either side. What is the probability that none of the ants meet?

- A) $\frac{1}{64}$ B) $\frac{1}{49}$ C) $\frac{1}{16}$ D) $\frac{1}{7}$ E) NOTA

18. A gardener wants to plant some colorful flowers in a row. They have four goldenrods, four lilacs, and three roses. We can assume each goldenrod is indistinguishable from the other goldenrods, and likewise for the lilacs and roses. The gardener plants the flowers at random. What is the probability that no two goldenrods are next to each other?

- A) $\frac{4}{165}$ B) $\frac{1}{22}$ C) $\frac{7}{33}$ D) $\frac{4}{5}$ E) NOTA

19. How many four-digit even numbers have all distinct digits?

- A) 2296 B) 2350 C) 2476 D) 2520 E) NOTA

20. An unfair coin has two sides, one with a value of 2 and the other with a value of 3. Let X represent the value of the side landing face up when the coin is flipped. For this coin, the expected value of X^2 is 7. What is the expected value of X^4 ?

- A) 49 B) 51 C) 55 D) 57 E) NOTA

21. Let S be the set of integers from 1 to 2026, inclusive. There is an n -element subset of S such that the sum of any two elements of the subset is not divisible by the difference of the same two elements. Find the largest possible value of n .

- A) 506 B) 507 C) 674 D) 675 E) NOTA

22. Twelve students, each of whom is a senior or junior, are at a math tournament. A teacher is to select three of those students for the tournament's team round such that at least one senior and at least one junior are selected. If 160 different such teams can be formed from the 12 students, which of the following can be the difference between the number of seniors and the number of juniors among these twelve students at the math tournament?

- A) 2 B) 4 C) 6 D) 8 E) NOTA

23. Three cards labeled 1, 2, and 3 are put in a bag. Three other cards also labeled 1, 2, and 3 are put in a second bag. A card is drawn from each bag, and the two cards are then stacked in a pile on the table. This is repeated two more times without replacement. What is the probability that at least one of the stacks will contain cards with the same number?

- A) $\frac{1}{6}$ B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) $\frac{2}{3}$ E) NOTA

24. Four darts are thrown at a 4×4 square that has been divided into sixteen 1×1 squares whose sides are parallel to the 4×4 square. If each dart lands in a different small square, what is the probability that the squares in which they land will form a horizontal, vertical, or diagonal line?

- A) $\frac{1}{4368}$ B) $\frac{1}{182}$ C) $\frac{1}{64}$ D) $\frac{5}{32}$ E) NOTA

25. Aiden and Billie each tell the truth with probability $\frac{1}{4}$ and lie with probability $\frac{3}{4}$. Aiden makes a statement, and Billie says that the statement made by Aiden is true. What is the probability that Aiden is actually telling the truth?

- A) $\frac{1}{16}$ B) $\frac{1}{10}$ C) $\frac{1}{7}$ D) $\frac{1}{4}$ E) NOTA

26. You have an 80% chance of selecting the letter for the correct answer choice on this question, based on your mathematical ability. However, since I think this is too high, I randomly select two answer choices and swap the letters that denote them without letting you change your answer. What is the probability that your answer is right after this swap?

- A) 40% B) 48% C) 50% D) 75% E) NOTA

27. How many ordered quintuples $(x_1, x_2, x_3, y_1, y_2)$ of positive integers satisfy the equation $(x_1 + x_2 + x_3)(y_1 + y_2) = 91$?

- A) 256 B) 396 C) 576 D) 1024 E) NOTA

28. A point is dropped randomly and uniformly on a line segment 2 units long, cutting the segment into two pieces. What is the expected value of the ratio of the length of the shorter piece to the length of the longer piece?

- A) $\frac{1}{3}$ B) $\frac{1}{\pi}$ C) $\ln 2$ D) $2\ln 2 - 1$ E) NOTA

29. A circle is divided into 7 sectors. Find the number of ways to color these sectors with 11 colors such that no two adjacent sectors have the same color. For this problem, “adjacent” means that the two sectors share a radius that divides them.

- A) 1,663,200 B) 1,771,561 C) 9,900,000 D) 9,999,990 E) NOTA

30. Uris, Val, Wan, Xie, Yola, and Zelda are practicing the Mu Relay by taking 2026 distinct practice relays (the number of distinct relays is 2026). Each of the 2026 practice Relays has six questions. The questions of a Relay must be worked in order, and once one problem from one Relay is finished, a problem from a different Relay can be worked. Each problem can be done by any of the six people, and the Relay as a whole can be the work of more than one person. Let N be the total number of ways in which the problems can be answered by the six people, and let M be the maximum integer such that 3^M divides N . Compute the sum of the digits of M .

- A) 21 B) 19 C) 22 D) 16 E) NOTA