



Speed Math

Test #603

Name: _____

ID Number: _____

School: _____

Division (circle one):

Mu Alpha Theta Sponsor

- _____ 1. How many distinct permutations exist for the letters in the word FORTWORTH?
- _____ 2. What is the probability that any two random positive integers a and b satisfy the equivalence $ab + a + b \equiv 2 \pmod{3}$?
- _____ 3. A path begins at the point $(0,0)$ and ends at the point $(4,4)$, and from any lattice point, one may walk in the xy -plane exactly one unit to the right or one unit up, with one exception: the path may not go through the point $(2,2)$. How many distinct such paths exist?
- _____ 4. Consider $9^2 + 4 \cdot 12^2 + 4^5$. What is this quantity's units digit?
- _____ 5. Consider $9^2 + 4 \cdot 12^2 + 4^5$. What is the greatest prime factor of this quantity?
- _____ 6. Let $f(x) = \frac{1}{1-x}$, where $x \neq 1$. Simplify $f^{2025}(x)$, 2025 copies of f composed with each other, in terms of x .
- _____ 7. Elise flips a standard fair coin repeatedly. What is the probability that she flips a head immediately followed by a tail (HT) at some point before she flips two tails in a row (TT)?
- _____ 8. Given that $\log x + \log y = 4$ and $(\log x)^3 + (\log y)^3 = 12$, find the value of $(\log x)(\log y)$.
- _____ 9. How many positive integers from 1 to 100, inclusive, are divisible by 4 or 6?
- _____ 10. In triangle ABC , points D and E lie on sides BC and AC , respectively. Segments AD and BE intersect at point P , segments BD and DC are equal in length, and segment AE is three times as long as segment EC . Find the ratio of the length of segment AP to the length of segment PD .
- _____ 11. Calculate: $\sum_{k=0}^9 ((10-k) \cdot 2^k)$
- _____ 12. The solutions to the equation $x^4 + x^2 + 1 = 0$ are plotted in the complex plane, and those points are the vertices of a quadrilateral. Find the longest side length of this quadrilateral.
- _____ 13. The infinite fraction $1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{2 + \dots}}}$, where the first part of each denominator alternates between 1 and 2, and each numerator is 1, can be written in the form $\frac{a + \sqrt{b}}{c}$, where a , b , and c are positive integers, and b is not divisible by the square of any positive integer greater than 1. Find the value of $a + b + c$.
- _____ 14. Given that $0 < a, b, c \leq \frac{1}{4}$, find the minimum value of the expression $\frac{a}{bc} + \frac{b}{ac} + \frac{c}{ab}$.
- _____ 15. How many 4-digit positive integers exist in which the units digit is equal to the sum of the other digits?
- _____ 16. Let a vertex of a regular hexagon and the midpoints of the two sides of that hexagon that meet at that vertex be the three vertices of a triangle. The area enclosed by that triangle is what fraction of the area enclosed by the hexagon?
- _____ 17. What are the last two digits of 19^{125} ?
- _____ 18. If $\vec{a} = \langle 4, 1, 2 \rangle$ and $\vec{b} = \langle 0, 5, 2 \rangle$, find the magnitude of the cross product $\vec{a} \times \vec{b}$.

- _____ 19. Isabella walks at a constant 3 miles per hour for the duration of a 100-mile race. Ten minutes after Isabella begins walking, Melissa begins running at a constant 6 miles per hour, but she takes a 5-minute break, in which she stops moving, after every mile. How many times will Melissa and Isabella have traveled the same amount in the race at the same moment in time?
- _____ 20. When rolling two fair, standard, six-sided dice, what is the expected value of the sum of the rolled faces?
- _____ 21. What is the greatest integer value of $\frac{x^3 + 8}{x^2 - 4}$ if x is an integer that is neither 2 nor -2 ?
- _____ 22. Find the range, written in interval notation, of the function $y = f(x) = -4x^2 + 12x - 1$.
- _____ 23. Find the area enclosed by the ellipse with equation $\frac{x^2}{4} + \frac{y^2}{9} = 4$.
- _____ 24. Chicken nuggets come in boxes of 5 or 6 nuggets. What is the largest whole number of nuggets that cannot be ordered exactly using any combination of 5- and 6-nugget boxes?
- _____ 25. Evaluate, where x is in degrees : $\sum_{x=1}^{90} \sin^2 x$