

Mu Alpha Theta 2026 – Theta 2-D Geometry – Answers

1. C
2. B
3. A
4. C
5. B
6. A
7. B
8. A
9. D
10. D
11. C
12. B
13. A
14. E (148)
15. C
16. D
17. D
18. B
19. D
20. B
21. C
22. B
23. A
24. D
25. A
26. C
27. C
28. D
29. A
30. C

Mu Alpha Theta 2026 – Theta 2-D Geometry – Solutions

1. $\angle B \cong \angle C$. $5x - 3 + 2(3x + 9) = 180 \rightarrow x = 15$. $m\angle C = 54$

2. each side = 12cm. $A = \frac{1}{2}(12)(6\sqrt{3}) = 36\sqrt{3} \text{ cm}^2$

3. $r = h$. Vol of a cylinder is $V = \pi r^2 h \rightarrow \pi r^3$. $\pi r^3 = 512\pi \rightarrow r = 8, h = 8$
 $SA = 2\pi rh + 2\pi r^2 \rightarrow 2\pi(8)(8) + 2\pi(8^2) = 256\pi$

4. WXYZ is an isosceles trapezoid. Constructing 2 heights from W and X to $\overline{ZY}$ divides the trapezoid up into 2 congruent 6-8-10 right triangles and a 6 by 4 rectangle. Use the Pyth Thm with sides 6 and 12 to solve for diagonal $\overline{WY}$. $WY = XZ = 6\sqrt{5}$. The sum of the 2 diagonals is $12\sqrt{5}$

Or

Use Ptolemy's Theorem for cyclic quadrilaterals: $(WX)(ZY) + (XY)(WZ) = (WY)(XZ)$

5. The 12 vertices of the dodecagon divide the circle into 12 30° arcs. Angle RDT intercepts 7 of these arcs, and $m\angle RDT = \frac{1}{2}(210) = 105$

6. Use Heron's Formula to find the area of the triangle: $s = \frac{1}{2}(7+8+9) = 12$ and

$A = \sqrt{12(12-7)(12-8)(12-9)} = 12\sqrt{5}$. The longest altitude is drawn to the shortest side. Use $A = \frac{1}{2}bh$ to solve

for h. $12\sqrt{5} = \frac{1}{2}(7)h \rightarrow h = \frac{24\sqrt{5}}{7}$

7. $C = 2\pi r$. The sum of the circumferences of all 10 circles is
 $2\pi + 4\pi + 6\pi + 8\pi + 10\pi + 12\pi + 14\pi + 16\pi + 18\pi + 20\pi = 110\pi$

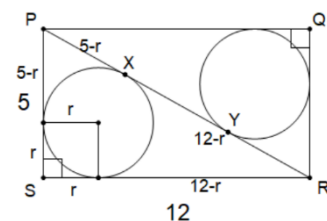
8. $48^2 + 56^2 > 72^2$, therefore the triangle is acute

9. Because of the angle bisector, $\frac{AC}{CD} = \frac{AB}{BD}$. $CD = 3, BD = 4$. To find the length of the angle bisector, $\overline{AD}$,
 $(AD)^2 + (CD)(BD) = (AC)(AB)$ and $(AD)^2 + (3)(4) = (6)(8) \rightarrow (AD)^2 = 36 \rightarrow AD = 6$

10. Triangle PRS and Triangle RQP are congruent 5-12-13 triangles.

Using the figure, $5 - r + 12 - r = 13 \rightarrow r = 2$

And $PX + XY + YR = 13 \rightarrow 3 + XY + 3 = 13 \rightarrow XY = 7$



11. Height of the cylinder = 2(radius of sphere) $\rightarrow h=2r$

$$\frac{\frac{4}{3}\pi r^3}{\pi r^2 \cdot 2r} = \frac{\frac{4}{3}}{2} = \frac{4}{6} = \frac{2}{3}$$

12. slope of $\overline{AB} = \frac{-1-3}{-4-1} = \frac{-4}{-5} = \frac{4}{5}$ slope of altitude $\perp$ to $\overline{AB} = -\frac{5}{4}$

Equation of line with slope of $-\frac{5}{4}$ passing through point C (-2, 5) is $y = -\frac{5}{4}x + \frac{5}{2}$

13. Let x be the angle. $180 - (90 - x) = 122$ $x = 32$

14. If $m\angle DBC = 16$, then $m\angle ABC = 32$. $\angle BCD$ is the supplement of $\angle ABC$, therefore $m\angle BCD = 148$

15. $AB = BC = CD = AD = DE = 6$ $m\angle BCD = (BC)^2 + (CE)^2 = (BE)^2 \rightarrow 6^2 + 12^2 = (BE)^2$ $BE = 6\sqrt{5}$

$\triangle BFA \cong \triangle EFD$ by AAS, therefore $EF = FB$ and $EF = \frac{1}{2}BE = \frac{1}{2} \cdot 6\sqrt{5} = 3\sqrt{5}$

16. Number of diagonals in an n -gon = $\frac{n(n-3)}{2}$ If $\frac{n(n-3)}{2} = 3915$, then $n = 90$. $\frac{360}{90} = 4^\circ$, measure of an exterior angle. The measure of one interior angle is $180 - 4 = 176^\circ$

17. method for finding area of a polygon on the coordinate plane:

(a) List the points in a traceable order, repeating the starting point

(b) Let D equal the sum of the downward diagonal products using the values in the ordered pairs

(c) Let U equal the sum of the upward diagonal products using the values in the ordered pairs

(d) Area = $\frac{1}{2}|D - U|$

(2,4) $D = (2)(-1) + (8)(-3) + (-3)(6) + (-6)(4) = -68$

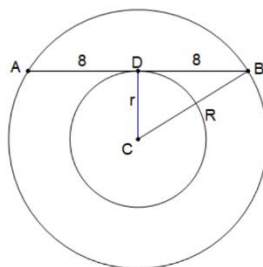
(8,-1) $U = (2)(6) + (-6)(-3) + (-3)(-1) + (8)(4) = 65$

(-3,-3)

(-6,6) $A = \frac{1}{2}|-68 - 65| = \frac{133}{2} = 66.5$

(2,4)

18. $r^2 + 8^2 = R^2 \rightarrow R^2 - r^2 = 8^2 = 64$
 $A = \pi R^2 - \pi r^2 = \pi(R^2 - r^2) = 64\pi$



19. The figure consists of a 45-45-90 triangle inside a 30-60-90 triangle

$$(2+x)^2 + x^2 = (2x)^2$$

$$4 + 4x + x^2 + x^2 = 4x^2$$

$$0 = 2x^2 - 4x - 4$$

$$0 = x^2 - 2x - 2$$

Using Quad Formula, $x = 1 + \sqrt{3}$ or $x = \sqrt{3} - 1$

$$20. {}_{100}C_3 = \frac{100!}{3!97!} = \frac{100 \cdot 99 \cdot 98}{3 \cdot 2 \cdot 1} = 161700$$

$$21. \frac{1}{3} \pi r^2 (6) = 8\pi \rightarrow r = 2 \quad r^2 + h^2 = l^2 \rightarrow 2^2 + 6^2 = l^2 \rightarrow l = 2\sqrt{10}$$

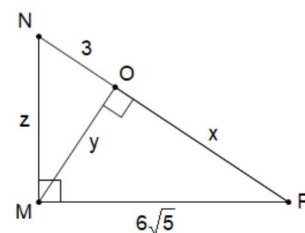
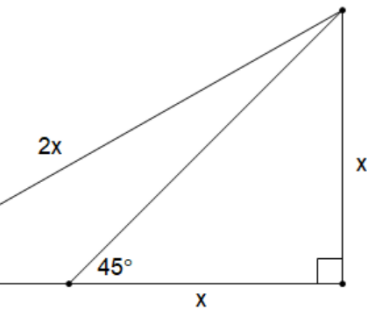
22. Three similar right triangles

$$\frac{x+3}{6\sqrt{5}} = \frac{6\sqrt{5}}{x} \rightarrow x = 12$$

$$\frac{3}{y} = \frac{y}{12} \rightarrow y = 6$$

$$x + y + z = 18 + 3\sqrt{5}$$

$$\frac{15}{z} = \frac{z}{3} \rightarrow z = 3\sqrt{5}$$



$$23. x^2 = (12+6)(6) \rightarrow x = 6\sqrt{3}$$

24. In the regular hexagon, the radius of the inner circle is the apothem of the hexagon, and the radius of the outer circle is the radius of the hexagon (distance from center to a vertex). These are sides of a 30-60-90

triangle and the ratio of the radius of the inner circle to the radius of the outer circle is $\frac{\sqrt{3}}{2}$. This is the scale

factor for the 2 circles, so the ratio of the areas of the circles is $\left(\frac{\sqrt{3}}{2}\right)^2 \rightarrow \frac{3}{4}$

$$25. \text{Triangle CAR is a 9-12-15 right triangle. } \sin R = \frac{12}{15} = \frac{4}{5} \text{ and } 2\cos C = 2\left(\frac{12}{15}\right) = \frac{8}{5} \quad \frac{4}{5} + \frac{8}{5} = \frac{12}{5}$$

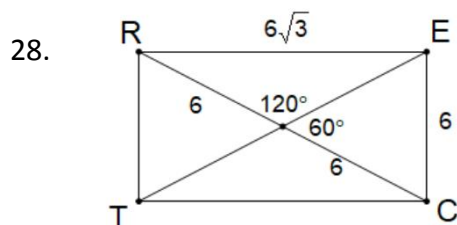
26. The centers of the 4 spheres form a tetrahedron with side length of 2. Using right triangles and

Pythagorean Theorem, the height of the tetrahedron is $\frac{2\sqrt{6}}{3}$ and the distance from a vertex to the centroid of

the tetrahedron is $\frac{3}{4} \cdot \frac{2\sqrt{6}}{3} = \frac{\sqrt{6}}{2}$. The radius of the largest sphere that could fit in the space between the given spheres is this distance minus the radius of one of the spheres $\frac{\sqrt{6}}{2} - 1 = \frac{\sqrt{6} - 2}{2}$

27. Circumference of circle = perimeter of square. If the perimeter of the square = $2\pi r$, each side of the square is equal to $\frac{\pi r}{2}$. The ratio of the area of the square to the area of the circle is

$$\frac{\frac{\pi r}{2} \cdot \frac{\pi r}{2}}{\pi r^2} = \frac{\pi}{4}$$

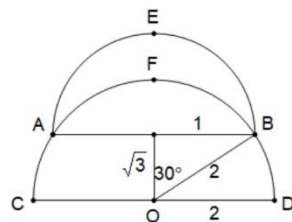


$$\text{Area of RECT} = 6(6\sqrt{3}) = 36\sqrt{3}$$

29. Area of region formed by AFB and $\overline{AB}$ = Area sector AFBO – Area triangle AOB

$$A = \frac{60}{360} \pi \cdot 2^2 - \frac{1}{2} \cdot 2 \cdot \sqrt{3} = \frac{2}{3} \pi - \sqrt{3}$$

$$\text{Area of semicircle with arc } AEB = \frac{1}{2} \pi 1^2 = \frac{\pi}{2}$$



$$\text{Area of lune} = \frac{\pi}{2} - \left(\frac{2}{3} \pi - \sqrt{3} \right) = \frac{\pi}{2} - \frac{2}{3} \pi + \sqrt{3}$$

$$\text{Area} = -\frac{\pi}{6} + \sqrt{3} = \frac{6\sqrt{3} - \pi}{6}$$

30. Scale factor of the two spheres is $\frac{2}{3}$. Ratio of volumes is $\frac{8}{27}$.

$$8x + 27x = 140\pi \rightarrow x = 4\pi$$

$$\text{Volume of larger sphere is } 108\pi. \quad 108\pi = \frac{4}{3} \pi r^3 \quad r = \sqrt[3]{81} = 3\sqrt[3]{3}$$