

Directions: For this test, E) NOTA means "None Of These Answers". The constant $i = \sqrt{-1}$. All units are assumed to be consistent. Where appropriate, you may use the approximations $\log(2) = 0.301$ and $\log(3) = 0.477$

1) Marc wants to find the perimeter of his favorite triangle, but only knows the length of each altitude. If the altitudes of his favorite triangle are 12, 15, and 20, what is the perimeter of his triangle?

- A) 60 B) 30 C) 47 D) $60\sqrt{3}$ E) NOTA

2) Marc wants an easier triangle question: He is tasked with finding the area enclosed by the triangle $\triangle ABC$. If A, B, C are all pairwise s units away from each other, what is the area enclosed by $\triangle ABC$?

- A) $s^2\sqrt{2}$ B) $s^2\sqrt{3}$ C) $2s^2$ D) $s^2\sqrt{5}$ E) NOTA

3) Jennifer is hanging an oddly shaped picture in an oddly shaped frame. Her frame is a circle of radius 4, and her picture is a regular octagon. If her picture fits perfectly within the frame (it is inscribed), what is the total area outside the picture but inside the frame?

- A) $32\pi - 16\sqrt{2}$ B) $16\pi - 32\sqrt{2}$
C) $48\pi - 24\sqrt{2}$ D) $24\pi - 48\sqrt{2}$ E) NOTA

4) Jeremy needs to find the sum of all the real zeros of the function below. He correctly identifies the answer, but then accidentally multiplies the correct answer by two! What is Jeremy's answer?

$$f(x) = (x - 1)(x + 2)(x^2 + 1)(x^4 - 2x^3 - 13x^2 - 4x - 30)$$

- A) -2 B) 0 C) 1 D) 2 E) NOTA

5) Arwen wants to list every single distinguishable permutation of the letters in the word ALGEBRAGEOMETRY on some lined notebook paper, one permutation per line. If each sheet contains 36 lines, how many sheets of paper will she need? Assume she can write on the front and the back of the paper.

- A) $\frac{15!}{3456}$ B) $\frac{15!}{72}$ C) $\frac{15!}{48}$ D) $15!$ E) NOTA

6) Mikaela must travel from the point $(2,5)$ to $(-4,3)$ while stopping along the x-axis before arriving. Then, she needs to get to $(8,2)$, again touching the x-axis before arriving. What is the minimum distance she can travel to accomplish this task?

- A) $2\sqrt{10} + \sqrt{145}$ B) $8 + 3\sqrt{5} + 3\sqrt{37}$ C) 23
D) Not Enough Information E) NOTA

7) Karah has three urns, three black pebbles, and three white pebbles. She places two black pebbles in the first urn and labels it BB, two white pebbles in the second urn and labels it WW, and the remaining two pebbles in the third urn and labels it BW. She leaves the room, and while she's gone, Logan comes and makes some swaps. He always keeps pebbles grouped as they were (meaning there's still a BB, BW, and WW pair after his swaps), and he always places a pair into an incorrectly labelled urn. When Karah comes back, she begins drawing pebbles one at a time from any urn. What is the minimum number of draws that she needs to make to determine the contents of all three urns?

- A) 0 B) 1 C) 2 D) 3 E) NOTA

8) Lillian is a bug-physicist, and needs to “determine the ant” by finding

$$\det \begin{pmatrix} 1 & 0 & 1 \\ 18 & 19 & 0 \\ 36 & 18 & 91 \end{pmatrix}$$

- A) 1729 B) -360 C) 1369 D) 2089 E) NOTA

9) The number of unique permutations of a standard $3 \times 3 \times 3$ Rubik's Cube is over 53 quintillion (5.3×10^{19}). Which of the following numbers is smaller than that? You may find the following useful: $\log(5.3) \approx 0.72$ and $20! \approx 2.43 \times 10^{18}$

- A) $11^{12^{13}}$ B) 20^{26} C) 6^{5^2} D) $22!$ E) NOTA

10) Piyush teaches a class in which a student's overall grade is the average of four equally weighted exams. It is given that the averages on each of the first three exams were 74, 81, and 79, there are 13 students in his class, and all students took exams 1, 2, and 3. If two students oversleep on the day of exam 4 (and make a 0 on that exam), what average should the remaining students achieve so that the overall class average is exactly 75?

- A) 68 B) 72 C) 75 D) 78 E) NOTA

11) Peter was solving polynomial equations for fun, and found a particular cubic that he thought was interesting! His polynomial $p(x)$ is a cubic with leading coefficient 1, and he noticed that the sum of the solutions to his polynomial was actually the product of the solutions to $p(1 - x)$. Given that $p(0) = 3$ and that no coefficients in $p(x)$ have magnitude larger than 4, what is the minimum value of the sum of the roots of $p(x)$?

- A) 4 B) 2 C) 0 D) -2 E) NOTA

12) Two polynomials are given to have the same number of candidate rational roots, as provided by the Rational Root Theorem. What can be said about the polynomials?

- A) The constant terms have the same number of factors.
B) The constant terms are both even or both odd.
C) Zero could be a root of either polynomial.
D) All of these answers.
E) NOTA

13) Jacob affixes points P and Q to the parabola $y - 2 = \frac{1}{4}(x - 3)^2$ at $(-5, 18)$ and $(11, 18)$ respectively. He then draws the two intersection points of the parabola with its latus rectum, labeling them R on the left and S on the right. If the intersection of PS and QR is at (m, n) , compute $\left(\frac{2m}{n}\right)!$

- A) 1 B) 24 C) 120 D) 5040 E) NOTA

14) Dallas (a person) travels to Dallas (the city in Texas) to watch Dallas (the 1950 Western Film). He drives from his hometown while listening to Dallas (the Jimmy Buffett song) on repeat the whole way. The song is 3 minutes and 25 seconds long, and in between each there's a 5 second delay before the song is repeated. If his drive is 14 hours, how many times does he listen to the Jimmy Buffett song?

- A) 120 B) 240 C) 360 D) 245.85 E) NOTA

15) For what values of k will the equation $kx^2 + (k - 2)x + \frac{2}{k} = 0$ have two real solutions?

- A) $k \in [2 - 2\sqrt{2}, 2 + 2\sqrt{2}]$
- B) $k \in (2 - 2\sqrt{2}, 2 + 2\sqrt{2})$
- C) $k \in (-\infty, 2 - \sqrt{2}] \cup [2 + 2\sqrt{2}, \infty)$
- D) $k \in (-\infty, 2 - 2\sqrt{2}) \cup (2 + 2\sqrt{2}, \infty)$
- E) NOTA

For questions 16 – 19, choose the answer choice that best describes the graph described in the question.

16) $2x^2 + 3y^2 - 5x = 2$, in the Cartesian plane

- A) Parabola
- B) Ellipse
- C) Circle
- D) Hyperbola
- E) NOTA

17) $|2x + 3y - 4z| = 4$, in Cartesian space

- A) Plane
- B) Ellipsoid
- C) Two Parallel Planes
- D) Cube
- E) NOTA

18) $2|x| + 3|y| + 4|z| = 6$, in Cartesian space

- A) Irregular Octahedron
- B) Irregular Hexahedron
- C) Irregular Dodecahedron
- D) Irregular Icosahedron
- E) NOTA

19) $|z - 2 - i| - |z + 4| = 7$, in the complex plane

- A) Circle
- B) Ellipse
- C) Parabola
- D) Hyperbola
- E) NOTA

For Questions 20 – 22, consider the following information: Jason, Christian and Griffen are all employees of a company that paints houses. They each paint at a constant rate, but that rate may not be consistent between jobs.

20) The painters must paint a house that contains 12 identical rooms. It takes Jason 3 hours to paint one of these rooms, Christian 5 hours to paint two of these rooms, and Griffen 7 hours to paint three of these rooms. How long (in hours) does it take to paint the whole house with all three working together and independently?

- A) between 10 and 10.5 B) between 10.5 and 11
C) between 9.5 and 10 D) between 9 and 9.5 E) NOTA

21) On a separate job, the crew must paint an apartment building that has three types of rooms. Room type A can be painted by Jason in 2 hours, Christian in 2.5 hours, and Griffen in 2.25 hours. Room type B can be painted by Jason in 1 hour, Christian in 0.75 hours, and Griffen in 1.25 hours. Room type C can be painted by Jason in 3 hours, Christian in 2 hours, and Griffen in 4 hours. If one person paints one type of room only, which configuration of painters to rooms results in the fastest rate of total painting?

- A) Jason paints B, Christian paints C, Griffen paints A
B) Jason paints C, Christian paints B, Griffen paints A
C) Jason paints B, Christian paints A, Griffen paints C
D) Jason paints C, Christian paints A, Griffen paints B
E) NOTA

22) Jason, Christian, and Griffen upload their consciousnesses to a machine that creates The Ultimate Painter (TUP). TUP is capable of painting any room at twice the maximum rate of any combination of the three subordinate painters. How long (in hours) would it take TUP to paint the apartment building in Problem 21 if there are 4 rooms of each type?

- A) between 2 and 3 hours B) between 3 and 4 hours
C) between 4 and 5 hours D) between 5 and 6 hours E) NOTA

23) How many total complex zeros, counting each instance for multiplicities greater than one, does polynomial $P(x)$ have, given that polynomials $Q(x)$ and $R(x)$ have degrees 4 and 7, respectively, and $P(x) = Q(x^2)R(x) + R(x^4)/Q(x^2)$?

- A) 15 B) 18 C) 21 D) 24 E) NOTA

24) Astryd is evaluating sums and needs some help evaluating the sum below:

$$\sum_{i=0}^{\infty} \frac{3}{x^2 + 5x + 4}$$

- A) 11 B) $\frac{11}{2}$ C) $\frac{11}{3}$ D) $\frac{11}{6}$ E) NOTA

25) Astryd wants to evaluate the sum below, but makes a critical mistake! She started from the incorrect index. If her answer is 10 less than the correct answer, what was her starting index? $\lfloor x \rfloor$ is the greatest integer less than or equal to x .

$$\sum_{i=1}^{1023} \lfloor \log_2 i \rfloor$$

- A) 2 B) 4 C) 8 D) 16 E) NOTA

26) Compute the sum

$$\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5}$$

- A) $\frac{163}{30}$ B) $\frac{163}{15}$ C) $\frac{163}{12}$ D) $\frac{163}{6}$ E) NOTA

27) Mike loves combinatorial identities! He uses his extensive knowledge on the subject to evaluate the sum below:

$$\sum_{k=0}^m (-1)^k \binom{r}{k} = (-1)^m \binom{r-1}{m}$$

If Mike chooses integers r and m that solve the system below, what is the value of his original sum?

$$\begin{aligned} m + r &= 17 \\ 5m + 12r &= 169 \end{aligned}$$

- A) 462 B) -462 C) 77 D) -77 E) NOTA

28) What is the smallest positive integer that has exactly 2026 positive integral factors?

- A) $2 \cdot 3^{1012}$ B) 2^{2026} C) $3 \cdot 2^{1013}$ D) $3 \cdot 2^{1012}$ E) NOTA

29) Sharanya writes the English alphabet a bunch of times in a row (in the correct order, duh). If we begin the index at 1, what letter lies at index 29^3 ? For those of you prone to phrasing-based disputes, the term “a bunch” is to be interpreted as “enough times to successfully complete this problem.”

- A) a B) w C) c D) k E) NOTA

30) Evaluate:

$$\left(\frac{45}{90} + \frac{138}{276} \right)^{2026}$$

- A) 1 B) 2 C) 3 D) ∞ E) NOTA