

1) A	2) E	3) B	4) D	5) A
6) C	7) B	8) C	9) C	10) D
11) C	12) E	13) A	14) B	15) D
16) B	17) C	18) A	19) D	20) A
21) B	22) D	23) E	24) D	25) C
26) E	27) B	28) D	29) A	30) A

1) The altitudes of a triangle are the heights from each base, but since $\frac{1}{2}b_i h_i = A \Rightarrow b_i h_i = 2A$. Let the sides of the triangle be x, y, z , then we have $12x = 15y = 20z = 2A$, which gives us $x = \frac{A}{6}$, $y = \frac{2A}{15}$, and $z = \frac{A}{10}$. Using Heron's formula, we note that $s = \frac{x+y+z}{2} = \frac{A}{5}$ and $A = \sqrt{s(s-x)(s-y)(s-z)} = \sqrt{\frac{A}{6} \cdot \frac{A}{30} \cdot \frac{A}{15} \cdot \frac{A}{10}} \Rightarrow A = \sqrt{\frac{A^4}{5^4 2^2 3^2}} \Rightarrow A = \frac{A^2}{150} \Rightarrow A = 150$. Since we know that $s = \frac{x+y+z}{2} = \frac{A}{5}$, we have $x + y + z = \frac{2A}{5} = \frac{2 \cdot 150}{5} = 60$

2) The area of an equilateral triangle with side s is $\frac{s^2\sqrt{3}}{4}$

3) Given the area of an octagon given the circumradius (from center to a vertex) is $A = 2\sqrt{2}R^2$, the area we search for is $A_{\text{circ}} - A_{\text{oct}} = \pi 4^2 - 2\sqrt{2} \cdot 4^2 = 16\pi - 32\sqrt{2}$

4) The real zeros will come from the $(x-1)(x+2)$ terms, and possibly the quartic. Using the Rational Root Theorem is a great way to guess a root, then you can synthetically divide it out to get a lower degree polynomial: RRT gives candidate roots $\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$. Testing out a few, we see that $x = -3, 5$ both work, with the remaining polynomial being irreducible over the reals. Therefore, the sum is $S = 1 - 2 - 3 + 5 = 1$, and Jeremy's answer is twice this $2S = 2$.

5) There are 15 letters in ALGEBRAGEOMETRY, and the repeats are A (2x), G (2x), E (3x), and R (2x). We need to divide this number by 72 (the number of permutations per sheet of paper), so our desired quantity is $\frac{15!}{2!2!3!2!72} = \frac{15!}{3456}$, which interestingly enough equals 378,378,000

6) We'll apply the classic trick of flipping over the x-axis twice. We reflect $(-4, 3)$ to $(-4, -3)$, and the distance to this point from $(2, 5)$ is $d = \sqrt{(5 - (-3))^2 + (2 - (-4))^2} = \sqrt{8^2 + 6^2} = 10$. Now we do the same for $(-4, 3)$ to $(8, 2) \rightarrow (8, -2)$ giving $d = \sqrt{(-4 - 8)^2 + (3 - (-2))^2} = \sqrt{12^2 + 5^2} = 13$. Therefore the total distance is $10 + 13 = 23$.

7) The key here is that Logan's switching puts pebble pairs in *incorrectly labeled* urns. Let's say that when Karah comes back in the room, she draws from the BB urn: if she draws a black pebble, she knows that this is the BW pair, then the BB pair is in the WW urn and the WW pair is in the BW urn. However, if she draws a white pebble, she doesn't know whether

she's pulling from BW or WW. A symmetric argument can be made for drawing from the WW urn. BUT, if she draws from the BW urn, the color she draws indicates with 100% certainty whether she pulled from the BB or WW pair; WLOG, let's say she pulls from the WW pair. Then the BB pair can't be in the BB urn, so must be in the WW urn, then the BW pair is in the WW urn. Thus, only one draw is necessary!

8) Using cofactor reduction, we have $\det = 1 \begin{pmatrix} 19 & 0 \\ 18 & 91 \end{pmatrix} - 0 \begin{pmatrix} 18 & 0 \\ 36 & 91 \end{pmatrix} + 1 \begin{pmatrix} 18 & 19 \\ 36 & 18 \end{pmatrix} = 19 \cdot 91 + 18^2 - 36 \cdot 19 = 1369$.

9) We can go down the line of answer choices and compare them to the target quantity, let's call $P = 5.3 \times 10^{19}$. Answer A is clearly larger than P, as $11^{12^{13}} > 10^{12^{13}} > 10^{20} > 5.3 \times 10^{19}$. Answer B is also much larger than P, since $20^{26} = 2^{26} \cdot 10^{26} > 5.3 \times 10^{19}$. Answer D is also larger, because $22! = 22 \cdot 21 \cdot 2.43 \times 10^{18} = 2.2 \cdot 2.1 \cdot 2.43 \times 10 \cdot 10 \cdot 10^{18} = 11.226 \times 10^{20}$. Answer C is smaller, and we can show this by taking the log of both quantities: $\log P = \log 5.3 + 19 \log 10 = 19.72$, and $\log C = 25 \log 6 = 25 \cdot (\log 2 + \log 3) = 25 \cdot 0.778 = 19.45$, showing that C is smaller.

10) The equation we want to solve is $75 = \frac{13 \cdot 74 + 13 \cdot 81 + 13 \cdot 79 + 11x}{52} \Rightarrow 3900 = 962 + 1053 + 1027 + 11x \Rightarrow 11x = 858 \Rightarrow x = 78$.

11) Let's write $p(x) = x^3 + ax^2 + bx + c$, which gives us $p(1-x) = (1-x)^3 + a(1-x)^2 + b(1-x) + c = -x^3 + (a+3)x^2 + (-3-2a-b)x + (a+b+c+1)$. The sum from $p(x)$ is $S = -\frac{a}{1} = -a$ and the product from $p(1-x)$ is $P = -\frac{a+b+c+1}{-1} = a+b+c+1$. If we set these two equal and solve for a , we see that $a = \frac{b+c+1}{-2}$. Using the given info that $p(0) = 3$, we have that $c = 3$ or that $a = \frac{-1}{2}b - 2$. If we plug that back into $p(x)$, we have $p(x) = x^3 + \left(-\frac{1}{2}b - 2\right)x^2 + bx + 3$. The sum of the solutions can now be written as $-\frac{-\frac{1}{2}b-2}{1} = \frac{1}{2}b + 2$. We want to minimize this expression given that all coefficients m satisfy $|m| \leq 4$. Thankfully, this minimization problem is easy, we have a minimum at the smallest allowable value of b , which is -4 , leading to a sum of solutions to be $\frac{1}{2}(-4) + 2 = 0$.

12) Each answer choice is false. Answer A and B are both contradicted by $f(x) = 2x^2 + 2$ having candidate roots $\pm 1, \pm 2, \pm \frac{1}{2}$ and $g(x) = x^2 - 9$ having candidate roots $\pm 1, \pm 3, \pm 9$ despite 2 and 9 having different numbers of factors, and opposite parity. Answer C is impossible since the Rational Root Theorem assumes the constant term of the given polynomial is non-zero, and the only way for 0 to be a factor of the polynomial is if the constant term is 0. Obviously D doesn't work either.

13) We need to find the focal distance for the parabola using the formula $f = \frac{1}{4a}$. Plugging in $a = \frac{1}{4}$, we have that $f = 1$, meaning the focus is only one unit up from the vertex (which is at (3,2)). To find the y-coordinate n , the lines through the two pairs of points have equations $y = 1.5x + 1.5$ and $y = -1.5x + 10.5$, which intersect at the point (3,6). The desired quantity is $\left(\frac{2 \cdot 3}{6}\right)! = 1! = 1$.

14) Each full iteration of the song + break takes 3 minutes and 30 seconds, or two full iterations takes 7 minutes. If we divide 14 hours by 7 minutes, we get 120 double-loops or a total of 240 plays of the song!

15) We must check the determinant $b^2 - 4ac > 0 \Rightarrow (k - 2)^2 - 4(k) \left(\frac{2}{k}\right) > 0 \Rightarrow (k - 2)^2 - 8 > 0 \Rightarrow k > 2 + 2\sqrt{2}$ or $k < 2 - 2\sqrt{2}$. Note that 0, which is not possible, is in neither interval.

16) Both variables are of degree 2, and the coefficients have the same sign. This is consistent with the standard form of an Ellipse

17) Without the absolute value signs, this is the equation of a single plane. With them there, the function becomes two-valued, as both (x, y, z) and $(-x, -y, -z)$ satisfy the equation, giving us two parallel planes

18) Again, we have a plane without the absolute value symbols. If we consider a simpler version of the problem $z = 2|x| + 3|y|$, we see a pyramid with diamonds as cross section, but since in our given function has an absolute value around the z , we again have a double-valued affair, giving us a double pyramid, also known as an Octahedron (irregular because not all the coefficients are equal).

19) The absolute value here is acting as the distance from a point z to the point in the complex plane that is within the absolute value bars. Since we have the difference of two distances being a constant, this is the definition of a hyperbola in the complex plane.

20) We need the fraction of one room that each painter can paint in one hour, and we get these by inverting the rates given in the problem: $J = 1/3$, $C = 2/5$, and $G = 3/7$. Together they can paint $J + G + C = 122/105$ of a room in one hour. Then we need x (a number of hours) such that $\frac{122x}{105} = 12 \Rightarrow x = 105 \cdot \frac{12}{122}$. We can see that $x < 105 \cdot \frac{12}{120} = 10.5$ and $x = \frac{1260}{122} > \frac{1260}{126} = 10$, so option A is the appropriate interval.

21) We want to minimize the time spent, so we want to maximize the amount of room painted in one hour. We can build a small table of room/hour rates, then explore all 6 possible sums.

Person \ Room	A	B	C
Jason	1/2	1	1/3
Christian	2/5	4/3	1/2
Griffen	4/9	4/5	1/4

For any given placement of Jason, there are two possible configurations of Christian and Griffen, giving us 6 possibilities. If Jason takes room A, then we have either $\frac{1}{2} + \frac{4}{3} + \frac{1}{4} = \frac{25}{12}$ or $\frac{1}{2} + \frac{4}{5} + \frac{1}{2} = \frac{9}{5}$. If Jason takes room B, then we have either $\frac{2}{5} + 1 + \frac{1}{4} = \frac{33}{20}$ or $\frac{4}{9} + 1 + \frac{1}{2} = \frac{35}{18}$. If Jason takes room C, then we have either $\frac{2}{5} + \frac{4}{5} + \frac{1}{3} = \frac{23}{15}$ or $\frac{4}{9} + \frac{4}{3} + \frac{1}{3} = \frac{19}{9}$. Only two of these options are larger than 2, so we can compare them and see that $\frac{25}{12} < \frac{19}{9}$, so the last option is best, corresponding to Jason taking room C, Christian taking room B and Griffen taking room A.

22) We need to find the max of each column above, so we just drop the lowest element and sum, then double, to get the table below:

	A	B	C
TUP	$2\left(\frac{1}{2} + \frac{4}{9}\right) = \frac{17}{9}$	$2\left(1 + \frac{4}{3}\right) = \frac{14}{3}$	$2\left(\frac{1}{2} + \frac{1}{3}\right) = \frac{5}{3}$

If we call a the time to paint all four A rooms, b the time to paint all four B rooms, and c to paint all four C rooms, then $\frac{17}{9}a = 4 \Rightarrow a = \frac{36}{17}$, $\frac{14}{3}b = 4 \Rightarrow b = \frac{6}{7}$, and $\frac{5}{3}c = 4 \Rightarrow c = \frac{12}{5}$. This gives the total painting time as $a + b + c = \frac{36}{17} + \frac{6}{7} + \frac{12}{5} = \frac{1260+510+1428}{595} = \frac{3198}{595}$ hours, or approximately 5.37 hours.

23) We know by exponentiation rules that $Q(x^2)R(x)$ has degree $2 \cdot 4 + 7 = 15$, and that $R(x^4)/Q(x^2)$ has degree $4 \cdot 7 - 2 \cdot 4 = 20$. That means the overall polynomial is of degree 20, so there are 20 complex roots. Note that even though we have a polynomial in the denominator, it is specified that the result is a polynomial (not rational!), so we can safely assume there is no remainder upon dividing $R(x^4)$ by $Q(x^2)$.

24) We need to rewrite the summand as the sum of two simpler fractions using partial fraction decomposition. If $\frac{3}{x^2+5x+4} = \frac{A}{x+1} + \frac{B}{x+4}$, we can multiply out the denominators to get $3 = A(x+4) + B(x+1)$. If we let $x = -1$, we get $3 = 3A \Rightarrow A = 1$ and if we let $x = -4$, we get $3 = -3B \Rightarrow B = -1$, so our original fraction can be written as $\frac{1}{x+1} - \frac{1}{x+4}$. In the summation, after the first three terms, everything will cancel out, leaving us just $\frac{1}{1} + \frac{1}{2} + \frac{1}{3} = \frac{11}{6}$.

25) The sum can be written out as $\lfloor \log_2 1 \rfloor + \lfloor \log_2 2 \rfloor + \lfloor \log_2 3 \rfloor + \lfloor \log_2 4 \rfloor + \lfloor \log_2 5 \rfloor + \lfloor \log_2 6 \rfloor + \lfloor \log_2 7 \rfloor + \lfloor \log_2 8 \rfloor + \dots = 0 + 1 + 1 + 2 + 2 + 2 + 2 + 3 + \dots = \sum_{i=1}^9 i \cdot 2^i$. If

Astryd is off by 10, but otherwise completes the sum appropriately, she must have left off the 0,1,1,2,2,2,2 terms, meaning she started at index 8.

26) Put all fractions over the common denominator of 60 to get $\frac{30}{60} + \frac{40}{60} + \frac{45}{60} + \frac{48}{60} = \frac{163}{60}$

$$27) \begin{aligned} m + r &= 17 \\ 5m + 12r &= 169 \end{aligned} \Rightarrow (r, m) = (12, 5) \Rightarrow \sum_{k=0}^5 (-1)^k \binom{12}{k} = (-1)^5 \binom{12-1}{5} =$$

$$- \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = -462$$

28) For a given prime factorization $n = p_0^{e_0} \cdot p_1^{e_1} \cdots p_k^{e_k}$, the number of factors is $(e_0 + 1)(e_1 + 1) \cdots (e_k + 1)$. We want this to equal $2026 = 2 \cdot 1013$, we can accomplish this with two primes p_0 and p_1 . Since we want to minimize this number, we'll choose the two smallest primes, 2 and 3. Then we'll assign the exponents $2 - 1 = 1$ and $1013 - 1 = 1012$ so that the overall number is smallest. Put the larger exponent on the 2, giving us $n = 2^{1013} \cdot 3$. Fun fact: the last digit of this number is 6.

29) You could multiply out 29^3 and then divide by 26, but there's a better way! We note that $29^3 = (26 + 3)^3 = 26^3 + 3 \cdot 3 \cdot 26^2 + 3 \cdot 3^2 \cdot 26 + 3^3$. Notice that the first three terms are multiples of 26, which means they fully cycle through one iteration of the alphabet, putting us right back at the beginning. So the only term we really need to worry about is the $3^3 = 27 = 26 + 1$, meaning we're back at position 1 after all that!

30) The fraction sum in the parenthesis simplifies to $\frac{1}{2} + \frac{1}{2} = 1$ so $1^{2026} = 1$