

All logarithms must be real-valued unless otherwise specified. Where applicable, “E) NOTA” indicates that none of the above answers is correct.

1. The expression $\frac{2026(2026^{2026})^{2026}}{(2026 \cdot 2026^{2026})^{2026}}$ can be written as 2026^k . Find the value of k .

- A) -4104675 B) -2025 C) -1 D) 4104675 E) NOTA

2. If $\log_{x+1} k = -1$ and $\log_k y = \frac{1}{4}$, where $k \neq 1$ is a positive real number, find x when $y = \sqrt{11}$.

- A) $-\frac{120}{121}$ B) $-\frac{10}{11}$ C) $-\frac{1}{11}$ D) $\frac{1}{\sqrt[10]{11}-1}$ E) NOTA

3. Find the product of all real solutions to the equation $2^{2x+1} - 2049 \cdot 2^x + 2^{10} = 0$.

- A) -1024 B) -512 C) -10 D) 1024 E) NOTA

4. Solve the equation: $\log_5(x+1) - \log_{25} x = 1$

- A) $\frac{1+2\sqrt{6}}{23}$ B) $\frac{\sqrt{5}}{5}$ C) $\frac{1+\sqrt{5}}{4}$ D) $\frac{23+5\sqrt{21}}{2}$ E) NOTA

5. Simplify the product: $20^{26} \cdot 26^{26} \cdot 20^{20} \cdot 26^{20}$

- A) 46^{46} B) 520^{46} C) 520^{520} D) 2026^{2026} E) NOTA

6. Evaluate: $\log_4 3 \cdot \log_5 4 \cdot \log_6 5 \cdots \log_{n+1} n \cdots \log_{81} 80$

- A) $\frac{1}{4}$ B) $2 + \log_4 5$ C) 4 D) 20 E) NOTA

7. Consider the elements of the set $\{2^{8000}, 3^{7000}, 4^{6000}, 5^{5000}, 6^{4000}, 7^{3000}, 8^{2000}\}$. If the elements are written in order from least to greatest, which element would be in the middle?

- A) 4^{6000} B) 5^{5000} C) 6^{4000} D) 7^{3000} E) NOTA

8. Using $\log 2 \approx 0.301$ and $\log 3 \approx 0.477$, approximate the value of $\log \frac{16}{225}$.

- A) -1.952 B) -1.148 C) -0.952 D) -0.748 E) NOTA

9. Solve: $3^{3^x} = 27^{81^9}$

- A) 27 B) 28 C) 36 D) 108 E) NOTA

10. Let S be the sum of the real solutions to $x^4 - 8x^3 + 25x^2 - 36x + 18 = 0$. Compute $\log_2 S$.

- A) 1 B) 2 C) 3 D) 4 E) NOTA

11. The equation $x^{x\sqrt{x}} = (x\sqrt{x})^x$ has two positive real solutions. What is the product of these two solutions?

- A) 1 B) $\sqrt{2}$ C) 2 D) $\frac{9}{4}$ E) NOTA

12. Consider the elements of the set $\{\log_2 21, \log_3 43, \log_5 24, \log_7 37, \log_9 97\}$. If the elements are written in order from least to greatest, which element would be in the middle?

- A) $\log_2 21$ B) $\log_3 43$ C) $\log_5 24$ D) $\log_7 37$ E) NOTA

13. What is the sum of the solutions to the equation

$$(\log_3 x)(\log_4 x)(\log_6 x) = (\log_3 x)(\log_4 x) + (\log_3 x)(\log_6 x) + (\log_4 x)(\log_6 x)?$$

- A) 1 B) 13 C) 37 D) 73 E) NOTA

14. Let $\lfloor x \rfloor$ be the greatest integer less than or equal to x . What integer n satisfies the

equation $\sum_{i=1}^n \lfloor \log_8 i \rfloor = 2026$?

- A) 869 B) 870 C) 871 D) 872 E) NOTA

15. Find the integer value of x that satisfies the equation $\log \left(\frac{(x)^{\frac{1}{x}}}{(x)^{\frac{1}{x+1}}} \right) = \frac{1}{5050}$.

- A) 10 B) 100 C) 101 D) 1000 E) NOTA

16. If $10^a = 4$, $10^b = 3$, and $10^x = 192$, then find x in terms of a and b .

- A) $3a + b$ B) $a^3 + b$ C) $10a^3b$ D) a^3b E) NOTA

17. All values of x for which $\log_{2026}(\log_{2024}(\log_{2022}(\log_{2020} x)))$ is defined and real are the values $x > c$. What is the value of c ?

- A) 2022^{2020} B) 2020^{2022} C) 2022^{2024} D) $2022^{2020^{2024}}$ E) NOTA

18. Let a and b be positive real numbers such that $a^b = b^a$ and $b = 8a$. Compute the value of a .

- A) $\sqrt[8]{7}$ B) $\sqrt[8]{8}$ C) $\sqrt[7]{7}$ D) $\sqrt[7]{8}$ E) NOTA

19. Find the sum of all real values of x that satisfy the equation $8^{\log_2 x} - 5 \cdot 4^{\log_2 x} = 2 \cdot 2^{\log_2 x} - 2^{\log_2 24}$.

- A) 3 B) 4 C) 5 D) 6 E) NOTA

20. How many digits does 2^{2026} have?

- A) 608 B) 609 C) 610 D) 611 E) NOTA

21. Mualphathetium is a radioactive substance which decays exponentially with time. In 2 hours, a sample of Mualphathetium will decay to 90% of its original mass. How many hours will it take for a sample of Mualphathetium to decay to 10% of its original mass?

- A) 18 B) $\frac{\ln 10}{\ln 10 - \ln 9}$ C) $\log_{\frac{9}{10}} 10$ D) $\frac{2}{1 - \log 9}$ E) NOTA

22. Suppose $47^{x \log_{23} 68} = 68^{2 \log_{13} 47}$, where $x = \log_b n$ for some positive integers b and n . Compute $b + n$, where b is as small as possible but greater than 10.

- A) 408 B) 442 C) 508 D) 542 E) NOTA

23. Compute the sum of all possible values of x that satisfy the equation $3^{x+1} - 18(3^{x-1})^2 = 1$.

- A) $\log_3 \frac{1}{2}$ B) $\frac{\sqrt[3]{4}}{8}$ C) 1 D) $\log_3 \frac{3}{2}$ E) NOTA

24. Compute the product of all possible values of x that satisfy the equation $(\log_x 4 - 4)(\log_2 x + 2) = -6$.

- A) $\frac{1}{8}$ B) $\frac{1}{2}$ C) 1 D) 2 E) NOTA

25. Let m and n be positive integers such that $3^m + 3^{m-1} = n$. What is the smallest value of n^m ?

- A) 2 B) 3 C) 4 D) 9 E) NOTA

26. Let n be an integer. What is the remainder when $5x^{2n+1} - 10x^{2n} + 3x^{2n-1} + 5$ is divided by $x+1$?

- A) -13 B) -3 C) 2 D) 4 E) NOTA

27. Which of the following answer choices is closest to the value of $\sum_{k=1}^{100} (\log(2^k))$?

- A) 1505.1 B) 1520.2 C) 2050.5 D) 3010.3 E) 3040.4

28. For what value(s) of k does the equation $k \cdot 2^x + 2^{-x} = 3$ have a single solution?

- A) $k = \frac{9}{4}$ B) $k = 0, \frac{9}{4}$ C) $k \leq 0$ D) $0 \leq k \leq \frac{9}{4}$ E) NOTA

29. Given that a and b are both greater than 1, $\sqrt[4]{72a} \cdot \sqrt[3]{315b}$ has a positive integral value. The smallest possible value of $a + b$ lies in which of the following intervals?

- A) $[250, 500)$ B) $[500, 1000)$ C) $[1000, 2000)$ D) $[2000, 4000)$ E) NOTA

30. What is the sum of all real x such that $(9^x - 27)^3 + (27^x - 9)^3 = (9^x + 27^x - 36)^3$?

- A) $-\frac{5}{6}$ B) $\frac{1}{6}$ C) $\frac{13}{6}$ D) $\frac{19}{6}$ E) NOTA