

MAO Nationals 2026

Theta Gemini

- 1) The expanding out 2026^2 gives a result of 4,104,676, so the sum of the digits of the expansion is equal to $4 + 1 + 0 + 4 + 6 + 7 + 6 = 28$ **(B)**
- 2) The total number of ways to rearrange the letters A, A, B, B, and C, assuming that the two As and the two Bs are distinguishable, is equivalent to the total number of ways to rearrange the 5 letters, which is equal to $5! = 120$. Out of those 120 rearrangements, the only way for the letters to be picked out in alphabetical order is if the two As are picked first, then the two Bs, then the remaining C. There are therefore two ways to rearrange the two As and two ways to rearrange the two Bs, giving us 4 possible ways to draw out the letters in alphabetical order. Therefore, the probability is equal to $\frac{4}{120} = \frac{1}{30}$ **(A)**
- 3) The formula for a circle in a cartesian plane is equal to $(x - w)^2 + (y - h)^2 = R^2$. Plugging in 3 for w , -2 for h , and 7 for R , we get $(x - 3)^2 + (y + 2)^2 = 7^2$, which, when you expand and simplify like terms, gives you $x^2 - 6x + y^2 + 4y - 36 = 0$ **(A)**
- 4) $4x^2 + 9y^2 - 48x + 72y + 144 = 0$ is equivalent to the equation $4x^2 - 48x + 9y^2 + 72y + 144 = 0$, which can then be written as $4(x^2 - 12x) + 9(y^2 + 8y) + 144 = 0$. Completing the square for both part, we then get $4(x^2 - 12x + 36) - 144 + 9(y^2 + 8y + 16) - 144 + 144 = 0$, which then gets simplified as $4(x - 6)^2 + 9(y + 4)^2 = 144$. Converting this into standard form, we get $\frac{(x-6)^2}{36} + \frac{(y+4)^2}{16} = 1$. Therefore, the area of the ellipse is $6 \times 4 \times \pi = 24\pi$ **(A)**
- 5) The quantity W is equal to 5625, the quantity X equal to 8000, the quantity Y equal to 5624, and the quantity Z equal to 4096. Therefore, the order from largest to smallest is $XWYZ$ **(C)**
- 6) The factors of 2026 are 1, 2, 1013, and 2026. Adding these up gives us 3042 **(D)**

7) By intuition, you want the side of length 2 to be opposite the 30° angle to maximize the area of the triangle. The height of the triangle will then be equal to $2\sqrt{3}$, and so the area of the triangle is equal to $2\sqrt{3}$ **(E)**

8) Let $\log_a 2 = x$ and $\log_9 b = y$. Then

$$\frac{1}{2x} + \frac{1}{y} = \frac{5}{2}$$

$$x + y = \frac{3}{2}$$

The top equation can be simplified as $\frac{y+2x}{2xy} = \frac{5}{2}$. We can multiply both sides by $2xy$ to get $y + 2x = 5xy$ and solve for y , giving us $y = \frac{2x}{5x-1}$. We then substitute this value into the second equation to get $x + \frac{2x}{5x-1} = \frac{3}{2}$ and simplify, giving us $\frac{5x^2+x}{5x-1} = \frac{3}{2}$. Multiplying both sides by $5x - 1$ yields $5x^2 + x = \frac{15}{2}x - \frac{3}{2}$. Solving this equation gives us $x = \frac{3}{10}, 1$. A quick observation shows us that $\frac{3}{10} = \log_a 2$ will not yield an integer solution for a , so $x = \log_a 2 = 1$, which gives us $a = 2$. This also gives us $y = \log_9 b = \frac{1}{2}$, which yields $b = 3$. Thus, $a + b = 2 + 3 = 5$ **(B)**

$$\begin{aligned} 9) & (3 + 6 + 9 \cdots + 198) + (11 + 22 + 33 + \cdots 198) - 2(33 + 66 + \cdots 198) = \\ & 3(1 + 2 + 3 + \cdots 66) + 11(1 + 2 + 3 + \cdots + 18) - 66(1 + 2 + \cdots 6) = \\ & 3\left(\frac{66 \cdot 67}{2}\right) + 11\left(\frac{18 \cdot 19}{2}\right) - 66\left(\frac{6 \cdot 7}{2}\right) = 6633 + 1881 - 1386 = 7128 \text{ **(B)}** \end{aligned}$$

10) Since the side length is $4\sqrt{2}$ and the exterior angles are 45° , we are either increasing or decreasing coordinates of vertices by 4 or $4\sqrt{2}$. It is simple enough to count out the units: $(4,0)$, $(0,4)$, then up $4\sqrt{2}$ to $C(0, 4 + 4\sqrt{2})$, then up and right 4 to $D(4, 8 + 4\sqrt{2})$, then right $4\sqrt{2}$ to $E(4 + 4\sqrt{2}, 8 + 4\sqrt{2})$. Therefore, $q - p = (8 + 4\sqrt{2}) - (4 + 4\sqrt{2}) = 4$. **(B)**

11) By adding up the five numbers, $715 + 1287 + 1716 + 1716 + 3003 = 8437$, which is equivalent to none of the answer choices **(E)**

12) The distance between the points $(-5, 3)$ and the line $5x - 12y = 17$ is 6, and the volume of the torus is equivalent to $2\pi \times 8\pi \times 6 = 96\pi^2$ **(B)**

13) We can use the Catalan-like Ballot Theorem, which says that number of sequences where $a \geq b$ is $\frac{a-b+1}{a+1} * \binom{a+b}{a}$. In this case, we say that $a = 6$ and $b = 4$, giving us $\frac{6-4+1}{6+1} \binom{6+4}{6} = 90$ **(D)**

14) The phrase DALLASTEXAS is 11 characters long, so if the phrase repeats every 11 characters. Therefore, the character that gets repeated is $1000 \bmod 11 = 10$, which is equivalent to A **(D)**

15) The parabola $y = (x - 2)(x + 4)$ can be FOILED into $y = x^2 - 2x + 4x - 8 = x^2 + 2x - 8$, which can then be expanded into vertex form into $y = (x + 1)^2 - 9$. Therefore, the vertex of the parabola is $(-1, -9)$ **(E)**

16) $\frac{A}{x-2} + \frac{B}{x+1} = \frac{A(x+1)+B(x-2)}{x^2-x-2} = \frac{(A+B)x+(A-2B)}{x^2-x-2} = \frac{0x+3}{x^2-x-2}$, therefore $A + B = 0$ **(E)**

17) We can use the Gaussian-Jordan method to augment the matrix with the 3x3 identity matrix, giving us $\begin{bmatrix} 1 & -1 & 8 & 1 & 0 & 0 \\ 1 & -4 & 4 & 0 & 1 & 0 \\ 0 & 3 & 2 & 0 & 0 & 1 \end{bmatrix}$. Row reducing this matrix gives

us $\begin{bmatrix} 1 & 0 & 0 & -\frac{10}{3} & \frac{13}{3} & \frac{14}{3} \\ 0 & 1 & 0 & -\frac{1}{3} & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 1 & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$, which means that the inverse matrix is $\begin{bmatrix} -\frac{10}{3} & \frac{13}{3} & \frac{14}{3} \\ -\frac{1}{3} & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$. **(B)**

18)

$$2c + 3g + 5j = 1.84$$

$$7c + 5g + 4j = 3.20$$

$$4c + 7g + 7j = 3.56$$

Using this system of equations, we get

$$4c + 6g + 10j = 3.68$$

$$(4c + 7g + 7j) - (4c + 6g + 10j) = g - 3j = -0.12$$

$$14c + 10g + 8j = 6.40$$

$$14c + 21g + 35p = 12.88$$

$$11g + 27p = 6.48$$

Combined this with $g - 3j = -0.12$, we get

$$9g - 27j = -1.08$$

$$20g = 5.40$$

$$g = 0.27$$

$$j = 0.13$$

$$2c + 3(0.27) + 5(0.13) = 1.84$$

$$2c + 0.81 + 0.65 = 1.84$$

$$2c = 0.38$$

$$c = 0.19$$

Therefore, $3c + 3g + 3j = 3(0.19) + 3(0.27) + 3(0.13) = 1.77$ **(D)**

19) By calculating the determinant of the 3 by 3 matrix, we get $-2y - 20 + 98 + 7y - 35 + 16 = 0$, which can then be simplified by $5y + 59 = 0$, so $y = -\frac{59}{5}$ **(E)**

$$20) \frac{1}{\ln(\sqrt{e})} \times \ln\left(\frac{1}{\sqrt{e}}\right) \times \ln(e^2) = \frac{1}{\ln(e^{\frac{1}{2}})} \times \ln\left(e^{-\frac{1}{2}}\right) \times \ln(e^2) = \frac{1}{\frac{1}{2}} \times \frac{-1}{2} \times 2 = -2$$

(D)

$$21) .6 \times .2 \times .8 + .4 \times .8 \times .8 + .4 \times .2 \times .2 = .096 + .256 + .016 = 0.368$$
 (D)

22) If the shorter base has a length of 10 and the circle has a radius of 17, then half of the shorter base and the radius of the circle forms a right triangle with

hypotenuse of 17 and a width of 5. Therefore, the length between the shorter base and the center is $\sqrt{289 - 25} = \sqrt{264} = 2\sqrt{66}$. The longer base has a distance 8 from the center of the circle, and therefore half the length of the longer base is 15, and therefore the length of the longer base is 30. The area of the trapezoid is therefore $(30 + 10)\frac{1}{2}(8 + 2\sqrt{66}) = 160 + 40\sqrt{66}$ **(C)**

23) The probability Mr. Smith has more boys than girls is equal to $\frac{1-P(6 \text{ boys})}{2}$, and the probability Mr. Smith has 6 boys is $\binom{12}{6} \cdot \frac{1}{2^{12}} = \frac{924}{2^{12}} = \frac{231}{1024}$. Therefore, the probability Mr. Smith has more boys than girls is $\frac{1 - \frac{231}{1024}}{2} = \frac{793}{2048}$ **(C)**

24) $A = 11 + 13 + 15 \cdots 99 = (\text{number of terms})(\text{average}) = (95)\left(\frac{11+99}{2}\right) = 45 \times 55 = 2475$. $B = 102 + 104 \cdots 200 = (\text{number of terms})(\text{average}) = (50)(151) = 7550$. Therefore $B - A = 7550 - 2475 = 5075$ **(B)**

25) $0.26666 \dots = \frac{2}{3} - \frac{2}{5} = \frac{10}{15} - \frac{6}{15} = \frac{4}{15}$ so $a = 4$ **(C)**

26) $(20 + A)(10A + B) = 1971 = 3^3 \times 73 = 27 \times 73$ so clearly $A = 7$ and $B = 3$. Therefore $A + B = 7 + 3 = 10$ **(C)**

27) Even 5-digit palindromes end in either 2, 4, 6, or 8, so there are 4 options for the last digit, since it can't start in 0. There are then 10 options for both the second digit and the third digit, so there is a total of $4 \times 10 \times 10 = 400$ **(C)**

28) There are $\binom{4}{2}$ ways that the first 4 games can go, as out of the first 4 games, Tom has to win 2. Therefore, the probability that Tom wins two games after 4 games is $\binom{4}{2} \times \frac{3}{5} \times \frac{3}{5} \times \frac{2}{5} \times \frac{2}{5}$. The probability that Tom wins the fifth game is $\frac{3}{5}$, so the probability Tom wins in exactly five games is $\binom{4}{2} \times \frac{3}{5} \times \frac{3}{5} \times \frac{2}{5} \times \frac{2}{5} \times \frac{3}{5} = \frac{648}{3125}$ **(B)**

29) The coefficient of the $x^2 y^8$ term is $\binom{10}{2} (2)^2 (-3)^8 = 45 \cdot 4 \cdot 6561 = 1180980$ **(E)**

30) By factorizing the quartic, this is equal to $(x + 5)(x + 2)(x - 4)(x - 5)$, so the roots are $-5, -2, 4$, and 5 . The sum of the absolute values of the roots is therefore $5 + 2 + 4 + 5 = 16$ **(A)**