

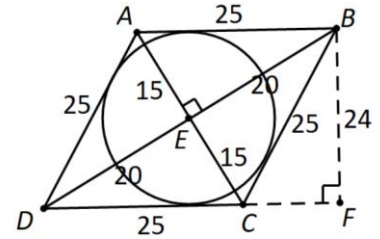
1. A A rhombus has 4 $\cong$ sides, diagonals that bisect each other, and $\perp$ diagonals.

$$\text{side} = \frac{100}{4} = 25$$

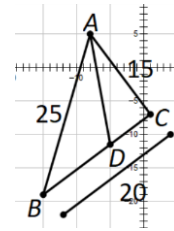
15, 20, 25 Pythagorean triple

$$\text{Area of rhombus} = \frac{(40)(30)}{2} = 600$$

$600 = 25h$; $h = 24$; The height of the rhombus is the diameter of the inscribed circle. The radius is 12. The area of the circle is $A = \pi 12^2 = 144\pi$.



2. C Use the distance formula to find $AB=25, AC=15$, and $BC=15$. Since $\overline{AD}$ is an angle bisector $\frac{25}{x} = \frac{15}{20-x}$; $x = \frac{25}{2}$. Therefore, $BD = \frac{25}{2}, CD = \frac{15}{2}$.



$$\text{Since } m\angle C = 90^\circ, AD = \frac{15\sqrt{5}}{2}$$

To find the coordinates for D , set up a system of equations using the distance formula to find AD , BD , and CD :

$$\left(\frac{15\sqrt{5}}{2}\right)^2 = (x+8)^2 + (y-5)^2, \left(\frac{15}{2}\right)^2 = (x-1)^2 + (y+7)^2, \left(\frac{25}{2}\right)^2 = (x+15)^2 + (y+19)^2$$

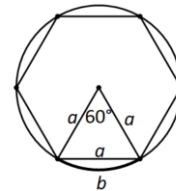
$D\left(-5, -\frac{23}{2}\right)$ The slope of $\overline{AD}$ is $-\frac{11}{2}$ and the equation for the angle bisector $\overline{AD}$ is

$$11x + 2y + 78 = 0, 11 + 78 = 89$$

3. B
$$\frac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{a^2} - \frac{1}{b^2}} = \frac{\frac{b+a}{ab}}{\frac{b^2-a^2}{a^2b^2}} = \frac{b+a}{ab} \cdot \frac{a^2b^2}{b^2-a^2} = \frac{b+a}{ab} \cdot \frac{a^2b^2}{(b+a)(b-a)} = \frac{ab}{b-a}$$

4. D
$$b = \frac{60^\circ \cdot 2\pi a}{360^\circ} = \frac{\pi a}{3}$$

$$a : \frac{\pi a}{3} = 1 : \frac{\pi}{3} = 3 : \pi$$



5. B $16y = (x-3)^2$, $y = \frac{1}{16}(x-3)^2$ The vertex is $(3,0)$, $a = \frac{1}{16}$ The length of the latus

rectum is 16. The focus is $\left(3, 0 + \frac{1}{4\left(\frac{1}{16}\right)}\right) = (3,4)$ So, the center of the circle is $(3,4)$

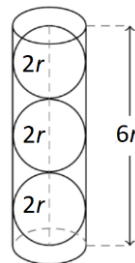
and the radius is 8. $(x-3)^2 + (y-4)^2 = 64 \rightarrow x^2 + y^2 - 6x - 8y - 39 = 0$

6. A Volume of tennis balls: $a = 3\left(\frac{4\pi r^3}{3}\right) = 4\pi r^3$

Volume of the cylinder: $\pi r^2(6r) = 6\pi r^3$

Volume of space: $b = 6\pi r^3 - 4\pi r^3 = 2\pi r^3$

$$4\pi r^3 : 2\pi r^3 \rightarrow 2:1$$



7. D ${}_{10}C_2(x^{10-2})(-2y)^2 \rightarrow \frac{10!}{8!2!}x^8(4y^2) \rightarrow 180x^8y^2$

8. A $r^2 = 15^2 + 15^2 - 2(15)(15)\cos 150^\circ$

$$r^2 = 450 + 225\sqrt{3} \rightarrow r = 15\sqrt{2 + \sqrt{3}}$$

$$\left(\sqrt{2 + \sqrt{3}}\right)^2 = \left(\sqrt{a} + \sqrt{b}\right)^2 \rightarrow 2 + \sqrt{3} = a + 2\sqrt{ab} + b \rightarrow 2 + \sqrt{3} = a + b + \sqrt{4ab}$$

$$2 = a + b \rightarrow a = 2 - b$$

$$3 = 4ab \rightarrow 3 = 4(2 - b)b$$

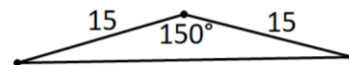
$$3 = 8b - 4b^2 \rightarrow 4b^2 - 8b + 3 = 0$$

$$(2b - 1)(2b - 3) = 0$$

$$b = \frac{1}{2}, b = \frac{3}{2}$$

$$\text{If } a = \frac{1}{2}, \text{ then } b = \frac{3}{2} \text{ and if } a = \frac{3}{2}, \text{ then } b = \frac{1}{2}. \sqrt{2 + \sqrt{3}} = \sqrt{\frac{1}{2}} + \sqrt{\frac{3}{2}} = \frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{2}$$

$$r = \frac{15\sqrt{2}}{2} + \frac{15\sqrt{6}}{2}, C = 2\pi\left(\frac{15\sqrt{2}}{2} + \frac{15\sqrt{6}}{2}\right), C = (15\sqrt{2} + 15\sqrt{6})\pi$$

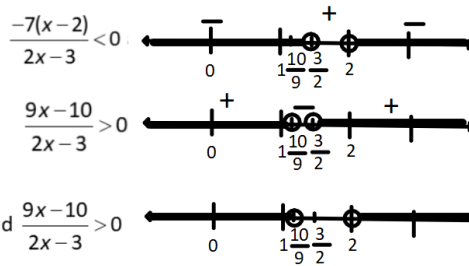


9. B $A = \frac{8(11)\sin 60^\circ}{2} = 22\sqrt{3}$

10. A $x = \frac{e^y}{5} \rightarrow 5x = e^y \rightarrow \ln(5x) = \ln(e^y) \rightarrow y = \ln 5 + \ln x$

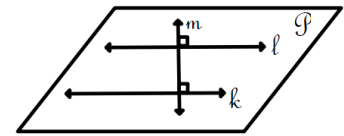
11. B $\frac{(x-3)(x^2+3x+9)}{(x+9)(x-4)} \cdot \frac{(x-4)(x+3)}{(x+3)(x-3)} = \frac{x^2+3x+9}{x+9}$

12. C $d_1^2 + d_2^2 = 2a^2 + 2b^2$, $a = 12, b = 5 \rightarrow d_1^2 + d_2^2 = 2(12)^2 + 2(5)^2 \rightarrow d_1^2 + d_2^2 = 338$



$$\left(-\infty, \frac{10}{9}\right) \cup \left(\frac{3}{2}, \infty\right)$$

19. D 15 per team, 45 total
 $(15)(3) - (10+9+11) + x = 22 \rightarrow 45 - 30 + x = 22 \rightarrow x = 7.$
20. C $\perp$ Transversal Theorem



21. C $x^2 + 2y^2 + 2xy + 6x + 2y$
 We can rearrange and regroup the terms into a quadratic expression with y as the main variable.

$$y^2(2) + y(2x+2) + (6x+x^2)$$

The lowest value will occur at

$$y = \frac{-b}{2a} = \frac{-2x-2}{2(2)} = \frac{-x-1}{2}$$

Substituting $\frac{-x-1}{2}$ for y

$$\left(\frac{-x-1}{2}\right)^2(2) + \left(\frac{-x-1}{2}\right)(2x+2) + (6x+x^2) \rightarrow \frac{x^2+2x+1}{2} - x^2 - 2x - 1 + 6x + x^2 \rightarrow$$

$$\frac{1}{2}x^2 + 5x - \frac{1}{2}$$

We want the minimum for this equation also.

$$x = \frac{-5}{2\left(\frac{1}{2}\right)} = -5. \text{ Substitute: } \frac{1}{2}(-5)^2 + 5(-5) - \frac{1}{2} = -13$$

22. B $10^{\log_x 10} = 10x^2 \rightarrow \log_{10} 10^{\log_x 10} = \log_{10} 10x^2 \rightarrow \log_x 10 = 1 + \log_{10} x^2 \rightarrow$

$$\frac{1}{\log_{10} x} = 1 + 2\log_{10} x \rightarrow 1 = \log_{10} x + \log_{10} x(2\log_{10} x) \rightarrow 1 = \log_{10} x + 2(\log_{10} x)^2$$

Let $u = \log_{10} x$

$$1 = u + 2u^2 \rightarrow 2u^2 + u - 1 = 0 \rightarrow u = \frac{1}{2}, u = -1$$

$$\frac{1}{2} = \log_{10} x \rightarrow 10^{\frac{1}{2}} = 10^{\log_{10} x} \rightarrow \frac{1}{2} = \log_{10} x \rightarrow 10^{\frac{1}{2}} \rightarrow x = \sqrt{10}$$

$$-1 = \log_{10} x \rightarrow 10^{-1} = x \rightarrow x = \frac{1}{10}$$

The product of the roots is $\sqrt{10} \cdot \frac{1}{10} = \frac{\sqrt{10}}{10}$.

23. D $AN \cdot NT = PN \cdot NS$ and $MN \cdot NB = PN \cdot NS$
So, $AN \cdot NT = MN \cdot NB$.

$$AN \cdot x = NB \cdot 2x \rightarrow \frac{AN}{NB} = \frac{2x}{x} \rightarrow \frac{AN}{NB} = \frac{2}{1}.$$

The radius of $\odot O$ is $\frac{2y+x}{2}$ and the radius of $\odot Q$ is

$$\frac{2x+y}{2} \cdot \frac{2y+x}{2} + \frac{2x+y}{2} - 3x = 324.$$

So, $y - x = 216$.

$$OP - PQ = \frac{2y+x}{2} - \frac{2x+y}{2} = \frac{y-x}{2}$$

$$\frac{y-x}{2} = \frac{216}{2} = 108.$$

24. B $\frac{6}{x} = \frac{x}{8} \rightarrow x^2 = 48 \rightarrow x = 4\sqrt{3}$.

25. C $2a + b = 20 \rightarrow b = 20 - 2a$

$$A = ab$$

$$A = a(20 - 2a) = -2a^2 + 20a$$

$$\text{Maximum: } x = \frac{-b}{2a} = \frac{-20}{2(-2)} = 5$$

$$A = -2(5)^2 + 20(5) = 50.$$

26. D The y-intercept of the equation $y = 2x - 6$ is $(0, -6)$. When $(0, -6)$ is reflected about the x-axis, the new point is $(0, 6)$.

27. D LPS's hammock is located at $(0, 20)$. Her pail is located at $B(-21, 8)$. To minimize the distance, reflect B over the x-axis. $B'(-21, -8)$. Use the distance formula to find that the distance from LPS to B is $3\sqrt{65}$ and the distance from B' to LPS is 35. Thus the shortest distance for LPS to go to B , then to the stream, and back to her original position is $35 + 3\sqrt{65}$.

28. B $\angle A \cong \angle E$ because both angles intercept CD and $\angle D \cong \angle C$ because both angles intercept AE . $\triangle ABD \sim \triangle EBC$ by AA. $\frac{8}{6} = \frac{x}{5} \rightarrow x = \frac{20}{3}$

