

1. If $A = \begin{bmatrix} -5 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \end{bmatrix}$, find $2A + 3B$.

- A. $\begin{bmatrix} -10 & 4 \end{bmatrix}$ B. $\begin{bmatrix} -2 & 2 \end{bmatrix}$ C. $\begin{bmatrix} -9 & 4 \end{bmatrix}$ D. $\begin{bmatrix} -7 & 4 \end{bmatrix}$ E. NOTA

2. If the determinant of 2×2 matrix A is 5, find $\det(2A)$.

- A. 25 B. 20 C. 10 D. 5 E. NOTA

3. Which of the following matrices is not in row-echelon form?

- A. $\begin{bmatrix} 1 & -3 & 2 \\ 0 & 1 & -6 \\ 0 & 0 & 0 \end{bmatrix}$ B. $\begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & -2 \end{bmatrix}$ C. $\begin{bmatrix} 1 & 7 & -2 \\ 0 & 1 & 5 \end{bmatrix}$ D. $\begin{bmatrix} 1 & -2 & 2 \\ 1 & -4 & -1 \\ 0 & 1 & 5 \end{bmatrix}$ E. NOTA

4. Find the sum of the elements in matrix B if $A = \begin{bmatrix} 3 & -2 \\ 1 & -4 \end{bmatrix}$ and $AB = \begin{bmatrix} -4 & -6 \\ 2 & -12 \end{bmatrix}$.

- A. 4 B. 0 C. 2 D. -6 E. NOTA

5. How many 3×3 matrices exist such that each entry is either a 5 or a 7?

- A. 9 B. 18 C. 27 D. 81 E. NOTA

6. Find the area enclosed by the parallelogram with vertices $(0, 0)$, $(2, 6)$, $(11, 8)$, and $(9, 2)$.

- A. 98 B. 100 C. 50 D. 49 E. NOTA

7. Which of the following left his law practice to become a full-time mathematician and later became a major contributor to matrix theory?

- A. Argand B. Boole C. Cayley D. Dirichlet E. NOTA

8. If $A = \begin{bmatrix} 1 & 2 & x \\ 3 & -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} y \\ x \\ 1 \end{bmatrix}$ and $AB = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$, then which of the following is true?

- A. $y = 2x$ B. $y = -2x$ C. $y = x$ D. $y = -x$ E. NOTA

9. Which of the following matrices cannot be obtained from the matrix $\begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$ by a single elementary row operation?

- A. $\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$ B. $\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$ C. $\begin{bmatrix} -1 & 2 \\ -2 & 7 \end{bmatrix}$ D. $\begin{bmatrix} -1 & 2 \\ -1 & 3 \end{bmatrix}$ E. NOTA

10. Let $A = \begin{bmatrix} 1+i & 1 \\ -i & 0 \end{bmatrix}$, where $i = \sqrt{-1}$. For how many values of n , $n \in \{1, 2, 3, \dots, 100\}$, does $A^n = A$?
- A. 25 B. 24 C. 20 D. 16 E. NOTA

11. Find the least value of the product abc for which $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} \geq 0$.
- A. -8 B. -1 C. $-4\sqrt{2}$ D. -4 E. NOTA

12. Let $i = \sqrt{-1}$. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + yi$, find x and y .
- A. $x=3, y=1$ B. $x=1, y=3$ C. $x=0, y=3$ D. $x=0, y=0$ E. NOTA

13. Let $\max(m, n)$ be the greater value of m and n , and let $\min(m, n)$ be the lesser value of m and n . If $m \neq n$, find the value of mn for the system of equations
- $$\begin{cases} \max(m, n) = 2m + 2n + 1 \\ \min(m, n) = m + 4n - 8 \end{cases}.$$
- A. -3 B. -28 C. -87 D. -171 E. NOTA

14. The matrix $\begin{bmatrix} 7 & 0 & 0 \\ 0 & 22 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is which type of matrix?
- A. identity B. symmetric C. skew-symmetric D. idempotent E. NOTA

15. If M and N are square matrices of the same order such that $MN = M$ and $NM = N$, find an expression necessarily equivalent to $MN^2 + N^2$.
- A. $M + N$ B. $2MN$ C. $2NM$ D. MN E. NOTA

16. At Matrix Market, all bananas are of equal price and all tomatoes are of equal price. Amber buys four bananas and four tomatoes for \$3.20, and Mary buys six bananas and two tomatoes for \$2.40. Find the positive difference in the price of one banana and one tomato.
- A. 10¢ B. 20¢ C. 40¢ D. 60¢ E. NOTA

17. Given the system of equations $x^2 + y^2 = m$ and $xy = n$, where m and n are positive numbers and x and y are real numbers, which of the following is a necessary and sufficient condition so that the system has at least one solution, (x, y) ?
- A. $m \geq 2n$ B. $2m \leq n$ C. $m \geq n^2$ D. $m \leq 2n^2$ E. NOTA

18. The solution(s) to $\begin{cases} \log_2(8x) = 1 + 2\log_2 y \\ 1 + \log_8 x = \log_8(10y - 12) \end{cases}$ can be written in the form (x, y) . Find the sum of all x and y .
- A. $\frac{17}{6}$ B. $\frac{33}{4}$ C. 824 D. $\frac{151}{39}$ E. NOTA
19. $372_{14} = X_7$. Find the base-10 sum of the digits of X .
- A. 24 B. 6 C. 12 D. 4 E. NOTA
20. Find the coefficient of xy^4 in the expansion of $(x + y^2)^8 \left(1 - \frac{1}{x^2 y}\right)^5$.
- A. 160 B. 360 C. 240 D. 560 E. NOTA
21. If $x = -1$ is a triple root of $f(x) = x^4 + ax^3 + bx^2 + cx + 2$, what is the value of a ?
- A. -3 B. -2 C. 2 D. 6 E. NOTA
22. If $x + y = M$ and $x^2 + y^2 = N$, find $x^3 + y^3$ in terms of M and N .
- A. $\frac{M^3 - N^2}{3MN}$ B. $\frac{M(3N - M^2)}{2}$ C. $\frac{M(N - 3M^2)}{2}$ D. $\frac{M(N^2 - 3M)}{2}$ E. NOTA
23. M and N are sets such that M has two more elements than N and 96 more subsets than N . How many elements are in M ?
- A. 3 B. 5 C. 7 D. 9 E. NOTA
24. A pair of lines is generated by the equations $6x^2 - xy - 12y^2 = 0$ and $15x^2 + 14xy - 8y^2 = 0$. Find an equation of a line parallel to—and 7 units away from—the line that is common to each of the given equations.
- A. $3x + 4y = 7$ B. $3x + 4y = -7$ C. $3x + 4y = -35$ D. $3x + 4y = 42$ E. NOTA
25. A positive number x can be written as $x = [x] + \{x\}$, where $[x]$ represents the greatest integer function and $\{x\}$ represents the fractional part. If it is known that $\begin{cases} a + [b] + \{c\} = 5.3 \\ b + [c] + \{a\} = 3.6, \\ c + [a] + \{b\} = 4.9 \end{cases}$ where a, b , and c are positive, find $\{b\}$.
- A. 0.8 B. 0.6 C. 0.5 D. 0.3 E. NOTA

26. Given that $\frac{xy}{x+y}=1$, $\frac{yz}{y+z}=\frac{1}{2}$, and $\frac{xz}{x+z}=\frac{1}{5}$, find the product xyz .

- A. $-\frac{1}{2}$ B. $-\frac{1}{3}$ C. $-\frac{1}{6}$ D. $\frac{1}{9}$ E. NOTA

27. For the functions $f(x)=\sqrt{x}$ and $g(x)=x^2-6x+9$, $y=f(g(x))$ and $y=mx$ do not intersect. Which of the following is a possible value of m ?

- A. -2 B. -1 C. 1 D. 0 E. NOTA

28. If $\begin{cases} a^2x+y+a=0 \\ (a+1)x+2y+2=0 \end{cases}$ has no solutions, find the possible value(s) of a .

- A. 1 only B. $\frac{1}{2}$ only C. 1 and $-\frac{1}{2}$ only D. -1 only E. NOTA

29. (See Figure 29 below.) In unit square $MATH$, E and N are the midpoints of $\overline{MH}$ and $\overline{HT}$, respectively. Find the area enclosed by quadrilateral $LINE$.

- A. $\frac{3}{8}$ B. $\frac{5}{16}$ C. $\frac{7}{24}$ D. $\frac{5}{24}$ E. NOTA

30. (See Figure 30 below.) If line segments $\overline{MH}$ and $\overline{AT}$ intersect at point O , what is the degree measure of $\angle MOT$?

- A. 50 B. 55 C. 60 D. 65 E. NOTA

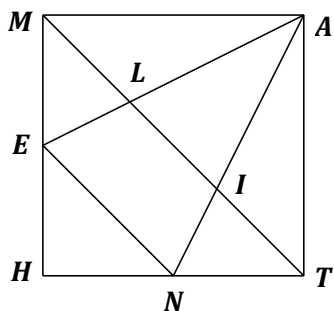


Figure 29

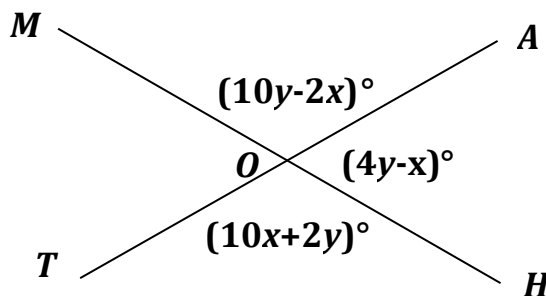


Figure 30