

0. **A:** 5, **B:** $\frac{576}{5}$, **C:** 13, **D:** π

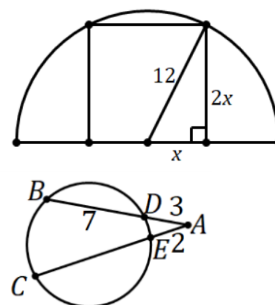
A: $B = \frac{5!}{1! \cdot 4!} = 5$

B: $4x^2 + x^2 = 12^2 \rightarrow x^2 = 144$. Area of the square is
 $4x^2 \rightarrow 4\left(\frac{144}{5}\right) = \frac{576}{5}$

C: $(3)(10) = (2)(CE) \rightarrow CE = 13$

D: $\sin^2 \theta = 6(\cos(-\theta) + 1) \rightarrow \sin^2 \theta = 6(\cos \theta + 1) \rightarrow 1 - \cos^2 \theta = 6\cos \theta + 6 \rightarrow$

$\cos^2 \theta + 6\cos \theta + 5 = 0 \rightarrow (\cos \theta + 5)(\cos \theta + 1) = 0 \rightarrow \cos \theta = -5, \cos \theta = -1 \rightarrow \theta = \pi$



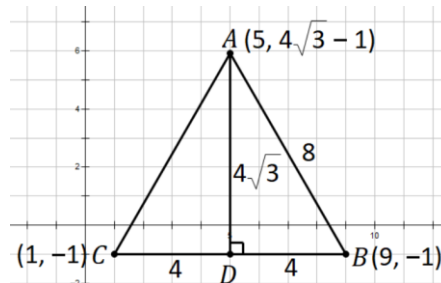
1. **A:** 2, **B:** $\frac{24}{13}$, **C:** 55, **D:** $A(5, 4\sqrt{3} - 1)$

A: $\log x + \log(x+2) = \log 8 \rightarrow \log(x^2 + 2x) = \log 8 \rightarrow x^2 + 2x - 8 = 0 \rightarrow x = 2$

B: $\frac{0.4 + x}{2 + x} = \frac{35}{100} \rightarrow 40 + 100x = 70 + 35x \rightarrow x = \frac{6}{13} \rightarrow \frac{6}{13} \text{ gal} \cdot \frac{4 \text{ qt}}{1 \text{ gal}} = \frac{24}{13} \text{ qt}$

C: $x = \frac{3}{5}y$ and $y - x < 24 \rightarrow y - \frac{3}{5}y < 24 \rightarrow y < 60$. Since x and y must be integers, the largest value for y that is less than 60 and makes x an integer is 55.

D: The sides of the equilateral triangle measure 8. Using the $30^\circ - 60^\circ - 90^\circ$ rules, the altitude of the triangle is $4\sqrt{3}$. So, the coordinates for A are $(5, 4\sqrt{3} - 1)$



2. **A:** $x = \pm 2$, **B:** $y = 0$, **C:** $(\pm 3, 0)$, **D:** $\left(0, \frac{9}{4}\right)$

A: To find the vertical asymptotes, factor the denominator and set the factors of the denominator equal to zero and solve.

$f(x) = \frac{(x+3)(x-3)}{(x^2+1)(x+2)(x-2)} \rightarrow x^2 + 1 = 0, x + 2 = 0, \text{ and } x - 2 = 0$. $x^2 + 1 = 0$ has no real solutions; so, the vertical asymptotes are $x = \pm 2$.

B: Since the degree of the denominator is larger than the degree of the numerator, the equation of the horizontal asymptote is $y=0$.

C: To find the x -intercepts, set the numerator equal to 0 and solve.

$$f(x) = \frac{(x+3)(x-3)}{(x^2+1)(x+2)(x-2)} \rightarrow x+3=0, x-3=0. \text{ So, the } x\text{-intercepts are } (\pm 3, 0).$$

D: To find the y -intercepts, substitute 0 in for x . $f(0) = \frac{(0+3)(0-3)}{(0^2+1)(0+2)(0-2)} = \frac{9}{4}$. So, the

$$y\text{-intercept is } \left(0, \frac{9}{4}\right).$$

3. A: $(0, \pm\sqrt{3})$, B: 50, C: $36\sqrt{3}$, D: 1

$$\mathbf{A:} \quad -(x-1)^2 - y^2 = -4 \rightarrow -x^2 + 2x - 1 - y^2 = -4 \rightarrow 4x = 0 \rightarrow x = 0$$

$$(x+1)^2 + y^2 = 4 \rightarrow x^2 + 2x + 1 + y^2 = 4 \rightarrow$$

$$(0+1)^2 + y^2 = 4 \rightarrow y = \pm\sqrt{3} \rightarrow (0, \pm\sqrt{3})$$

B. The diagonals of a square are $\perp$ to each other and bisect each other. The diagonals both measure 10.

$$A = \frac{10 \cdot 10}{2} = 50.$$

C: Draw the diagram. Construct $\overline{AE}$ so that $\overline{AE} \perp \overline{CD}$.

Since the radii are $\perp$ to the common external tangent segment at the points of tangency, $AEDC$ is a rectangle.

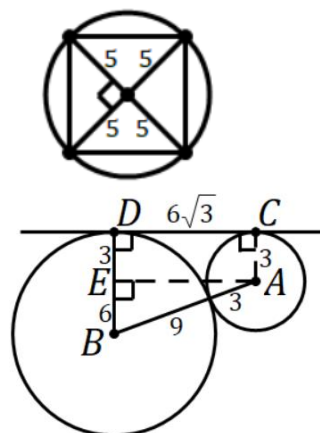
Use the Pythagorean Theorem to find $AE = 6\sqrt{3} = CD$.

$ABDC$ is a trapezoid. The area is $\frac{(3+9)(6\sqrt{3})}{2} = 36\sqrt{3}$.

$$\mathbf{D:} \quad x^2 - 4x + 1 = 0 \rightarrow x = 2 \pm \sqrt{3} \rightarrow (2 + \sqrt{3})(2 - \sqrt{3}) = 1$$

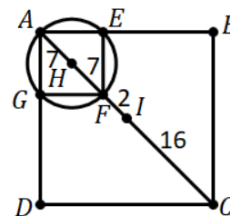
4. A: $y=3x-1$, B: 49π , C: $\frac{2}{195}$, D: $-i+2j+7k$

A: The midpoint of the segment is $(1, 2)$. The slope of the segment is $-\frac{1}{3}$. The slope of the $\perp$ is 3. Using the point $(1, 2)$ and the slope 3, the equation of the $\perp$ bisector of the segment is $y=3x-1$.



B: $\frac{AC \cdot BD}{2} = 512 \rightarrow \frac{x^2}{2} = 512 \rightarrow x = 32$. $AC = 32$, $CI = AI = 16$.

Since $FI = 2$, $AH = FH = 7$. The area of $\odot H$ is $\pi 7^2 = 49\pi$.



C: $\binom{15}{4} = \frac{15!}{4!11!} = 1365$ There are 1365 way to choose 4 points from 15 points. There will be 3 2×2 squares, 8 1×1 squares, and 3 tilted squares for a total of 14 squares.

$$\frac{14}{1365} = \frac{2}{195}.$$

D:

$$\begin{array}{ccccc} \hat{i} & \hat{j} & k & \hat{i} & \hat{j} \\ 3 & -2 & 1 & 3 & -2 \end{array} \rightarrow (\hat{i} \cdot 2 \cdot 1 + \hat{j} \cdot 1 \cdot -1 + k \cdot 3 \cdot 3) - (-1 \cdot -2 \cdot k + 3 \cdot 1 \cdot \hat{i} + -1 \cdot 3 \cdot \hat{j}) = -\hat{i} + 2\hat{j} + 7k$$

$$\begin{array}{ccccc} -1 & 3 & -1 & -1 & 3 \end{array}$$

5. **A:** $\frac{28}{155}$, **B:** $f^{-1}(x) = \sqrt[5]{x^3 + 2}$, **C:** $\frac{35\sqrt{74}}{74}$, **D:** 24π

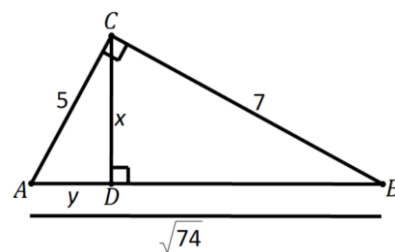
A: Ella saves $\$500 \cdot 0.20 = \100 on her bicycle. The original price for Ella's helmet is $x - x(0.10) = 108 \rightarrow x = \120 . So, Ella saves \$12 on her helmet. The fraction of Ella's savings on her total purchase is $\frac{100 + 12}{50 + 120} = \frac{28}{155}$

B: $x = \sqrt[3]{y^5 - 2} \rightarrow x^3 = y^5 - 2 \rightarrow f^1(x) = \sqrt[5]{x^3 + 2}$

C: Use the Pythagorean Theorem to find $AB = \sqrt{74}$.

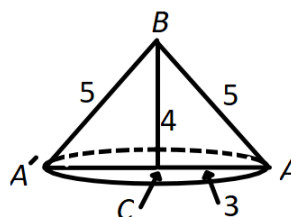
$$\frac{\sqrt{74}}{5} = \frac{5}{y} \rightarrow y = \frac{25}{\sqrt{74}}.$$
 Use the Pythagorean

Theorem to find $CD = \frac{35\sqrt{74}}{74}$.



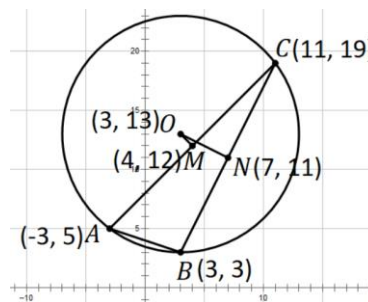
D: The resulting figure is a right cone with a radius of 3, a height of 4, and a slant height of 5.

$$SA = \pi 3^2 + \pi 3 \cdot 5 = 24\pi$$



6. A: $(3,13)$, B: $6\sqrt{5}$, C: $-2 \pm 2\sqrt{3}$, D: $\frac{26}{5}$

A: The center of the circle is the circumcenter of the triangle. The circumcenter is the intersection of the $\perp$ bisectors of the triangles. The midpoint of $\overline{AC}$ is $M(4,12)$ and the slope of $\overline{AC}$ is 1. The slope of the $\perp$ bisector of $\overline{AC}$ is -1 and the equation of the $\perp$ bisector is $y = -x + 16$. The midpoint of $\overline{BC}$ is $N(7,11)$ and the slope of $\overline{BC}$ is 2. The slope of



the $\perp$ bisector of $\overline{BC}$ is $-\frac{1}{2}$ and the equation of the $\perp$ bisector is $y = -\frac{1}{2}x + \frac{29}{2}$. The solution to the system of equations is $O(3,13)$, the center of the circle.

B: Use synthetic division to find that one of the solutions of the equation $x^3 - 20x^2 + 127x - 252 = 0$ is $x = 4$. Then, factor to find that the other two solutions are $x = 7$ and $x = 9$. The sides of the triangle are 4, 7, and 9. The semiperimeter of the triangle is $\frac{4+7+9}{2} = 10$ so the area is $A = \sqrt{10(10-4)(10-7)(10-9)} = 6\sqrt{5}$.

C: $(-2x)(x^2) - (-16)(3x) = 64 \rightarrow x^3 - 24x + 32 = 0$. Use synthetic division to find that $x = 4$ then use the quadratic formula to find the other two solutions $x = -2 \pm 2\sqrt{3}$. The irrational values for x are $-2 \pm 2\sqrt{3}$.

D: $ax^2 + bx + c = y$

$$(0,2) \rightarrow a(0)^2 + b(0) + c = 2 \rightarrow c = 2$$

$$(-4,5) \rightarrow a(-4)^2 + b(-4) + c = 5, (8,-12) \rightarrow a(8)^2 + b(8) + c = -12$$

Substituting $c = 2$ and solving the system of equations

$$16a + 4b = 3 \text{ and } 64a + 8b = -4, \text{ find } a = -\frac{5}{16} \text{ and } b = 2.$$

$$-\frac{5}{16}x^2 + 2x + 2 = y. \quad x = \frac{-2}{2 \cdot -\frac{5}{16}} = \frac{16}{5}. \text{ The maximum value for the parabola is}$$

$$-\frac{5}{16}\left(\frac{16}{5}\right)^2 + 2\left(\frac{16}{5}\right) + 2 = \frac{26}{5}.$$

7. **A:** $\frac{-x^2-2x}{4}$, **B:** $\frac{x^2}{25} + \frac{y^2}{45} = 1$, **C:** $\frac{41}{20}$, **D:** $\sqrt{13} \pi$

A: $(f \circ g)(x) = -\left(-\frac{x}{2}\right)^2 + \left(-\frac{x}{2}\right) = \frac{-x^2-2x}{4}$.

B: $e = \frac{c}{a} \rightarrow \frac{2}{3} = \frac{c}{a}$. $c^2 = a^2 - b^2 \rightarrow 3^2 - b^2 = 2^2 \rightarrow b^2 = 5$. We know that $b = 5$, so $b^2 = 25$.

This means that $a^2 = 45$. The equation is $\frac{x^2}{25} + \frac{y^2}{45} = 1$.

C: Construct $\overline{CF} \perp \overline{AE}$. $ABCF$ is a rectangle. $AB = 2 = CF$.

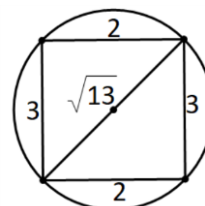
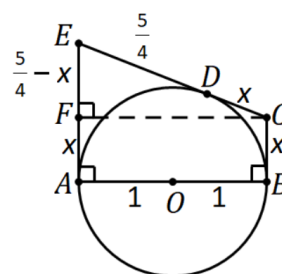
$AE = \frac{5}{4} = DE$ and $AF = BC = CD = x$. Use the

Pythagorean Theorem to find x .

$2^2 + \left(\frac{5}{4} - x\right)^2 = \left(\frac{5}{4} + x\right)^2 \rightarrow x = \frac{4}{5}$. The area of trapezoid

$ABCE$ is $\frac{\left(\frac{4}{5} + \frac{5}{4}\right) \cdot 2}{2} = \frac{41}{20}$.

D: Use the Pythagorean Theorem to find the diameter of the circle is $\sqrt{13}$. The circumference of the circle is $C = \sqrt{13} \pi$.



8. **A:** $-128 + 128\sqrt{3}i$, **B:** $\frac{15}{32}$, **C:** $\frac{121\sqrt{2}}{4}$, **D:** $5\sqrt{3}$

A: $\binom{4}{0}(2)^4 + \binom{4}{1}(2)^3(-2i\sqrt{3}) + \binom{4}{2}(2)^2(-2i\sqrt{3})^2 + \binom{4}{3}(2)(-2i\sqrt{3})^3 + \binom{4}{4}(-2i\sqrt{3})^4 =$

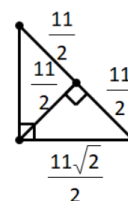
$16 - 64i\sqrt{3} - 288 + 192i\sqrt{3} + 144 = -128 - 128\sqrt{3}i$.

B: $\sum_{k=1}^4 \left(\frac{1}{2}\right)^{k+1} = \left(\frac{1}{2}\right)^{1+1} + \left(\frac{1}{2}\right)^{2+1} + \left(\frac{1}{2}\right)^{3+1} + \left(\frac{1}{2}\right)^{4+1} = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} = \frac{15}{32}$

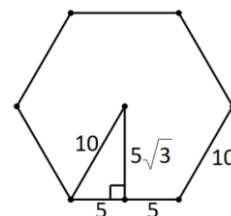
C: Use the $45^\circ - 45^\circ - 90^\circ$ to find that $a = \frac{11\sqrt{2}}{2}$. The midpoint

of a right triangle is equidistant from the vertices of the

triangle. So, $b = \frac{11}{2}$. $a \cdot b = \frac{11\sqrt{2}}{2} \cdot \frac{11}{2} = \frac{121\sqrt{2}}{4}$

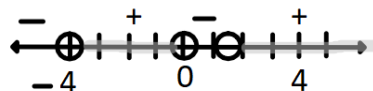


D: $\frac{3s^2\sqrt{3}}{2} = 150\sqrt{3} \rightarrow s = 10$. The radius of the hexagon is the same length as the side. Use the $30^\circ-60^\circ-90^\circ$ rules to find that the apothem measures $5\sqrt{3}$.



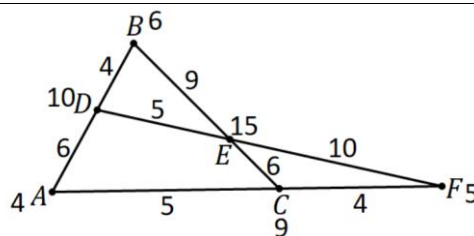
9. A: $(-4, 0) \cup \left(\frac{3}{2}, \infty\right)$, **B:** -287 , **C:** $\frac{1}{2}$, **D:** 100°

A: $f(x) = 2x^3 + 5x^2 - 12x \rightarrow x(2x-3)(x+4) = 0$
 $x = 0, x = \frac{3}{2}, x = -4$. The positive regions are
 $(-4, 0) \cup \left(\frac{3}{2}, \infty\right)$



B: $a_1 = 7, d = -3$ To find the 99th term, $a_{99} = 7 + (99-1)(-3) = -287$.

C: Construct the diagram. $\frac{DE}{EF} = \frac{1}{2}$.



D: $55^\circ = \frac{70^\circ + mAB}{2} \rightarrow mAB = 40^\circ$ and $65^\circ = \frac{40^\circ + mAG - 30^\circ}{2} \rightarrow mAG = 120^\circ$.

$m\angle ACE = 125^\circ$, so $125^\circ = \frac{120^\circ + 30^\circ + mBD}{2} \rightarrow mBD = 100^\circ$.

10. A: $\frac{15\sqrt{26}}{26}$, **B:** 608π , **C:** $\frac{\sqrt{2}}{6}$, **D:** 0.46

A: The distance between the point $(-1, 3)$ and the line $5x - y - 7 = 0$ is

$$\frac{|5(-1) - 1(3) - 7|}{\sqrt{25 + 1}} = \frac{15\sqrt{26}}{26}.$$

B: The volume of the frustum is $\frac{\pi \cdot 6}{3}(12^2 + 8^2 + 12 \cdot 8) = 608\pi$.

C: $f\left(-\frac{\pi}{4}\right) + 2f\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right)$ Let $y = f\left(-\frac{\pi}{4}\right)$ and $x = f\left(\frac{\pi}{4}\right)$ and $\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$.

$$f\left(-\frac{\pi}{4}\right) + 2f\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$f\left(\frac{\pi}{4}\right) + 2f\left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$y + 2x = \frac{\sqrt{2}}{2}$$

$$x + 2y = \frac{\sqrt{2}}{2}$$

Solve the system of equations for x : $x = \frac{\sqrt{2}}{6}$

$$D: P(\text{both win or both lose}) = (0.6)(0.3) + (0.4)(0.7) = 0.46$$

$$11. A: (5,17), B: 2\sqrt{7}, C: 4, D: \begin{bmatrix} \frac{2}{11} & -\frac{1}{11} \\ \frac{5}{22} & \frac{3}{22} \end{bmatrix}$$

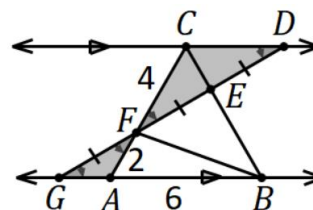
A: $11+6 > x$ and $x+6 > 11$ So the possible side lengths of the third side of the triangle are in the interval $(5,17)$.

B: $\triangle AFG \sim \triangle CFD$ by AA and the scale factor is $\frac{1}{2}$. Since

$$AC = 6, AF = 2 \text{ and } CF = 4. m\angle BAF = 60^\circ.$$

$$BF = \sqrt{2^2 + 6^2 - 2(2)(6)\cos(60^\circ)} = 2\sqrt{7}$$

(points E and F have been switched from the diagram on the test to this diagram on the solutions – the solution is valid though)



$$C: \sqrt{4-2\sqrt{3}} = \sqrt{\sqrt{3}^2 - 2\sqrt{3} + 1^2} = \sqrt{(\sqrt{3}-1)^2} = \sqrt{3}-1 \rightarrow a=3 \text{ and } b=1 \rightarrow 4$$

$$D: A^{-1} = \frac{1}{(3)(4) - (-5)(2)} \begin{bmatrix} 4 & -2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} \frac{2}{11} & -\frac{1}{11} \\ \frac{5}{22} & \frac{3}{22} \end{bmatrix}$$

$$12. A: 2, B: -\frac{17}{15}, C: 945, D: \frac{5\sqrt{6}}{2}$$

$$A: 2x + y + 3z = 12$$

$$7x + z = 31$$

$$2x + y + 3z = 12$$

$$x + z = 7$$

$$3x - 2y - 5z = 7$$

$$x + 3y + 4z = 1$$

$$7x + z = 31$$

$$x = 4$$

$$z = 3$$

$$y = -5$$

$$x + z = 7$$

$$x + y + z = 2$$

B: Use synthetic division with the solution $x = -4$ to find that the remainder is

$$20 + 15k. \text{ Solve for } k \quad 10 + 15k = 3 \rightarrow k = -\frac{17}{15}.$$

C: The polygon has 45 sides. $\frac{360^\circ}{n} = 8^\circ \rightarrow n = 45$. It has $\frac{45(45-3)}{2} = 945$ diagonals.

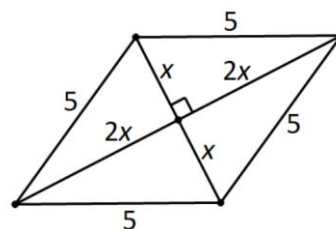
D: The ratio of the areas of the similar polygons is $\frac{2}{3}$. The ratio of the sides is

$$\frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{6}}{3}. \text{ So, } \frac{\sqrt{6}}{3} = \frac{5}{x} \rightarrow x = \frac{5\sqrt{6}}{2}.$$

13. **A:** $-\frac{2}{5}$, **B:** $6\sqrt{5}$, **C:** $\frac{\pi-2}{2}$, **D:** $\frac{11}{4}$

A: $5x^2 + kx - 4 = 0$. Since 2 is a solution, $x - 2$ must be a factor. Therefore,
 $(x - 2)(5x + 2) = 0 \rightarrow x = -\frac{2}{5}$

B: If the perimeter of the rhombus is 20, each side measures 5. Use the Pythagorean Theorem to find that $x = \sqrt{5}$. The shorter diagonal measures $2\sqrt{5}$ and the longer diagonal measures $4\sqrt{5}$. The sum of the diagonals is $6\sqrt{5}$.

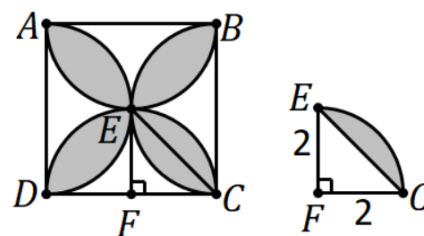


C: Construct the apothem of the square and the radius of the square. Find the area of the segment of the circle shown to the right.

$$\frac{90^\circ \pi 2^2}{360^\circ} - \frac{2 \cdot 2}{2} = \pi - 2$$

There are 8 congruent segments of circles; so, the area of the shaded region is $8\pi - 16$.

The probability that a point selected at random lies in the shaded region is $\frac{8\pi - 16}{16} = \frac{\pi - 2}{2}$.



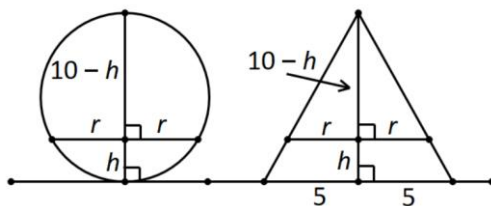
D: $5^{2(-2x+3)} = 5^{-5} \rightarrow x = \frac{11}{4}$

14. **A:** 2, **B:** $\frac{117}{2}$, **C:** $(n-2)!$, **D:** $[-2, -1) \cup (1, 5]$

A: To the right is a cross section of the sphere and of the right cone. In the circle,

$$(10-h)(h) = (r)(r) \rightarrow 10h - h^2 = r^2$$

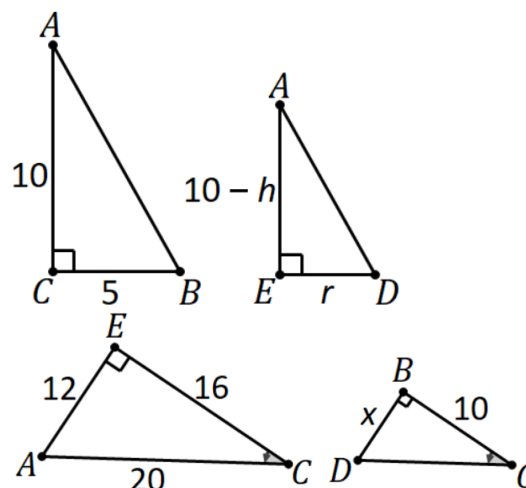
$$\triangle ABC \sim \triangle ADE$$



$$\frac{10}{10-h} = \frac{5}{r} \rightarrow h = 10 - 2r$$

$$r^2 = 10(10 - 2r) - (10 - 2r)^2$$

$$5r^2 - 20r = 0 \rightarrow r = 4, h = 2$$



B. $\triangle ACE \sim \triangle DCB$ by AA

$$\frac{16}{10} = \frac{12}{x} \rightarrow x = \frac{15}{2}$$

$$\text{The area of } \triangle ACE = \frac{12 \cdot 16}{2} = 96$$

$$\text{The area of } \triangle DCB = \frac{\frac{15}{2} \cdot 10}{2} = \frac{75}{2}$$

$$\text{The area of quadrilateral } ABDE \text{ is } 96 - \frac{75}{2} = \frac{117}{2}.$$

C: $\frac{(n)(n+1)(n)(n-1)!}{(n-1)!} \cdot \frac{(n)(n-1)!}{(n+1)(n-1)} = \frac{n^3(n-1)!}{n^3(n-1)} = \frac{n^3(n-1)(n-2)!}{n^3(n-1)} = (n-2)!$

D: $\frac{(x-5)(x+2)}{(x+1)(x-1)} \leq 0$
 $[-2, -1) \cup (1, 5]$

