

Important Instructions for this Test: Assume all questions ask for an exact answer unless otherwise specified. Questions are of varying difficulty, so it is recommended that you don't spend too long on any single question! Good luck, have fun, and as always: "NOTA" stands for "None of These Answers is correct."

1. Evaluate $\sum_{n=0}^{2026} \left(\frac{n}{1013} - 1 \right)$.

- A: -2 B: -1 C: 0 D: 1 E: NOTA

2. Evaluate $\prod_{n=2}^{31} \log_n(n+1)$.

- A: 5 B: 6 C: $5 \log_{31} 2$ D: $\log_{32} 31$ E: NOTA

3. Let $a_n = 2n - a_{n-1}$ where $a_0 = 4$. Find a_5 .

- A: 8 B: 0 C: 10 D: 2 E: NOTA

4. In the sequence defined in question 3, what value must a_0 take for a_4 and a_5 to be equal?

- A: -1 B: 1 C: 3 D: 5 E: NOTA

Use the Following Information for Questions 5 to 7:

Uncle Ben has a collection of infinitely many circles of radii $r_n = \frac{8}{3^n}$ where $n \geq 0$.

5. Uncle Ben places his circles externally tangential such that their centers are all collinear. What is the minimum length a piece of very thin string must be such that it can wrap around the edge of all the circles?

- A: 12π B: 24π C: 36π D: 48π E: NOTA

6. What is the total area of all of Uncle Ben's circles?

- A: 9π B: 96π C: 57.6π D: 72π E: NOTA

7. Now, Uncle Ben takes each circle and places their centers at the same point, forming a dartboard pattern. If a point is randomly and uniformly selected from within the largest circle, what is the probability that the smallest circle it lies within has an even index?

- A: $1/2$ B: $4/5$ C: $9/10$ D: $3/4$ E: NOTA

8. Suppose x_n is an arithmetic sequence with $x_2 = 4$ and $x_{20} = 26$. What is x_{29} ?

- A: 37 B: $\frac{322}{9}$ C: $\frac{344}{9}$ D: 35 E: NOTA

Use the Following Sequence for the Next 4 Questions:

Define P_n to be the sequence of primes in increasing order where $P_1 = 2, P_2 = 3$, etc.

9. What is P_{12} ?

- A: 37 B: 41 C: 39 D: 31 E: NOTA

10. Which of the following are not part of the sequence P_n ?

- A: 97 B: 127 C: 163 D: 193 E: NOTA

11. Let k be the smallest index such that $P_k > 100$. What is the remainder when P_k is divided by k ?

- A: 1 B: 3 C: 5 D: 23 E: NOTA

12. In number theory, it is known that the number of primes less than some integer N approaches $\frac{N}{\ln N}$ as N grows larger and larger. If $a = P_n$ is sufficiently large, which of the following is the best estimate for a in terms of n ? Recall that $\lfloor x \rfloor$ is the greatest integer (floor) function.

- A: $\lfloor n / \ln n \rfloor$ B: $\lfloor n * \ln n \rfloor$ C: $\lfloor \ln n \rfloor$ D: $\lfloor \ln n / n \rfloor$ E: NOTA

13. Evaluate $\sum_{n=0}^{13} (4n + 3|2n - 14|)$.

- A: 112 B: 658 C: 70 D: 322 E: NOTA

14. Let a_n be an arithmetic sequence with common difference d and $a_6 = 2$. If $\sum_{n=1}^6 a_n = 42$, find $\sum_{n=1}^6 \frac{d^n}{6}$.

- A: 21 B: $64/9$ C: 7 D: 5 E: NOTA

15. What is the 2026th digit to the right of the decimal in the expansion of $1/7$?

- A: 1 or 2 B: 3 or 4 C: 5 or 6 D: 7 or 8 E: NOTA

16. Find the sum of the following series: $\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \cdots + \frac{1}{\sqrt{2024}+\sqrt{2025}}$

- A: $\sqrt{2024} - 1$ C: 44 E: NOTA
B: $\sqrt{2024} + 1$ D: 45

17. Find the sum of the fourth and fifth terms in the quadratic sequence whose first, second, and third terms are 15, 20, and 33, respectively.

- A: 137 B: 105 C: 124 D: 143 E: NOTA

18. Evaluate the sum of the following infinite series: $\frac{2}{3} + \frac{8}{9} + \frac{24}{27} + \frac{64}{81} + \frac{160}{243} + \cdots + n \left(\frac{2}{3}\right)^n + \cdots$

- A: 2 B: 3 C: 6 D: 9 E: NOTA

Use the Following Information for the Next Two Questions:

In Statistics, an important property of a sample of data is known as the *sample variance* of that data. If the data is labeled as $x_1, x_2, \dots, x_n$ and the sample average of the data is denoted as $\bar{x}$, then the sample variance of this data, denoted by S_x^2 , is $S_x^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$.

19. Suppose $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ is a sample of data. Compute the sample variance on this sample.

- A: 10/3 B: 15/2 C: 20/3 D: 15/4 E: NOTA

20. Suppose that a sample of data with 16 values has sample variance $S_x^2 = 12$ and $\sum_{i=1}^{16} x_i^2 = 228$. What is the absolute value of the sample mean, $\bar{x}$, of this sample?

- A: 3 B: $\sqrt{2}$ C: $\sqrt{3}$ D: 2 E: NOTA

21. Compute $2 + \frac{2}{4 + \frac{1}{2 + \frac{1}{4 + \frac{1}{\dots}}}}$

- A: $\sqrt{6}$ B: $\sqrt{2}$ C: $1 + \frac{\sqrt{6}}{2}$ D: $1 - \frac{\sqrt{6}}{2}$ E: NOTA

22. Suppose G_n is a real-valued geometric sequence with $G_2 = 12$ and $G_5 = 4$. Evaluate $\sum_{n=1}^{\infty} G_{5n}$.

- A: $\frac{36}{9 + \sqrt[3]{3}}$ B: $\frac{36}{9 - \sqrt[3]{3}}$ C: $\frac{12\sqrt[3]{9}}{\sqrt[3]{3}-1}$ D: $\frac{12\sqrt[3]{9}}{\sqrt[3]{3}+1}$ E: NOTA

23. Evaluate $\sum_{n=5}^{24} \left(\frac{n+2}{n-1} * \binom{n+1}{3} \right)$.

- A: 17515 B: 14935 C: 17550 D: 16675 E: NOTA

24. What is the sum of the digits of the smallest index n such that the n th term of the arithmetic progression beginning $-16, -15.6, -15.2, \dots$ is greater than 2026? Assume that the arithmetic progression is 1-indexed (i.e., -16 is the first term with index 1, -15.6 is the second term with index 2, etc.).

- A: 9 B: 10 C: 11 D: 12 E: NOTA

25. Find the fractional part of $2026 + 202.\overline{6} + 20.\overline{26} + 2.0\overline{26}$.

- A: $\frac{443}{500}$ B: $\frac{10498}{10989}$ C: $\frac{43}{45}$ D: $\frac{38}{45}$ E: NOTA

26. Evaluate $34 * \sum_{n=1}^{\infty} (2^{-n} \prod_{m=1}^n i^m)$, where $i = \sqrt{-1}$.

- A: $-\frac{3}{128} + \frac{1}{32}i$ C: $-\frac{17}{120} + \frac{17}{30}i$ E: NOTA
B: $-6 + 8i$ D: $-\frac{1}{5} + \frac{2}{5}i$

27. How many trailing 0's does $2026!$ have when the resulting integer is written in base 72?

- A: 1734 B: 1010 C: 578 D: 505 E: NOTA

Consider the Following Information for the Last Three Questions:

In terms of recursive sequences, none are more famous than the Fibonacci sequence, which obeys the relation $F_n = F_{n-1} + F_{n-2}$ with $F_0 = 0$ and $F_1 = 1$. A related sequence is the Pell sequence (or Pell numbers), which obey the recursive relation $P_n = 2P_{n-1} + P_{n-2}$ with $P_0 = 0$ and $P_1 = 1$. Like the Fibonacci numbers, the Pell numbers have a closed form formula: $P_n = \frac{(1+\sqrt{2})^n - (1-\sqrt{2})^n}{2\sqrt{2}}$.

28. A third definition of the Pell numbers is possible using matrices. Namely, $\begin{bmatrix} P_{n+1} & P_n \\ P_n & P_{n-1} \end{bmatrix} = M^n$ for some matrix M . Which of the following is M ?

- A: $\begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix}$ B: $\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$ C: $\begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$ D: $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ E: NOTA

29. The Pell numbers are useful as they allow for an efficient rational approximation of $\sqrt{2}$. Namely one can approximate $\sqrt{2}$ by $\frac{P_{n+1}-P_n}{P_n}$. Using this approximation, which of the following is not an approximation of $\sqrt{2}$ using Pell numbers?

- A: 1.5 B: $\frac{577}{408}$ C: $\frac{17}{12}$ D: $\frac{100}{71}$ E: NOTA

30. Additionally, the Pell numbers can be used to form infinitely many Pythagorean triples. The legs for each right triangle have lengths $a_n = 2P_nP_{n+1}$ and $b_n = P_{n+1}^2 - P_n^2$, respectively. Let c_n be the sequence of lengths of the hypotenuse of the right triangle with legs a_n and b_n . Then, the ratio $\frac{c_{n+1}}{c_n}$ will approach some real number K as n grows larger and larger. What is K ?

- A: $1 + \sqrt{2}$ B: $3 + 2\sqrt{2}$ C: $\frac{1}{2} + \frac{\sqrt{2}}{4}$ D: $3 - 2\sqrt{2}$ E: NOTA