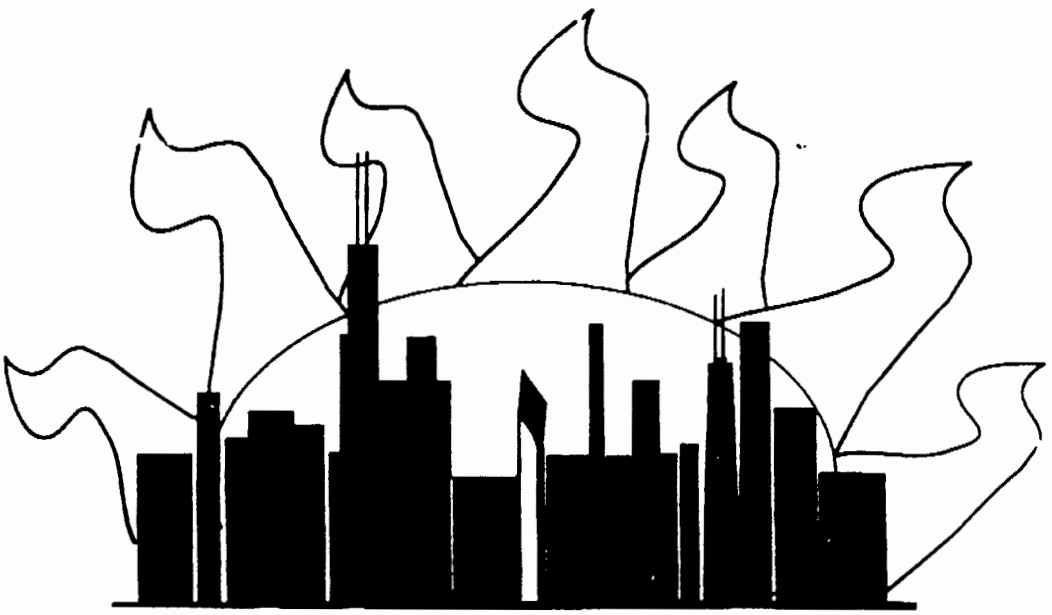
Alpha Division

School Bowl



Mu Alpha Theta National Convention
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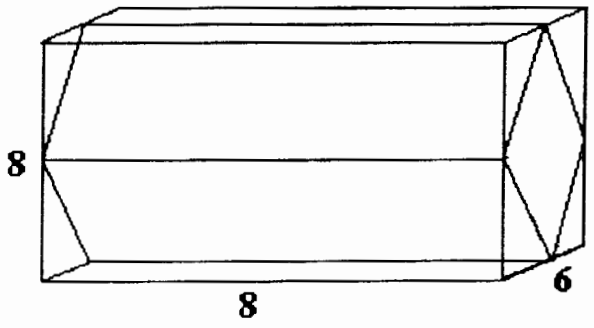
1. A positive integer can be written uniquely in the form $2^x 3^y 5^z k$, where k is a positive integer not divisible by 2, 3, or 5; and x, y, z are non-negative integers. Express the number 756,000 in this manner. Find $\frac{(k+x)}{yz}$ for this number.

2. A rectangular prism is inscribed within another rectangular prism. The lengths of the edges of the larger prism are as shown in the diagram below. If the vertices of the smaller prism bisect the edges of the larger prism, what is the difference between the volumes of the prisms?

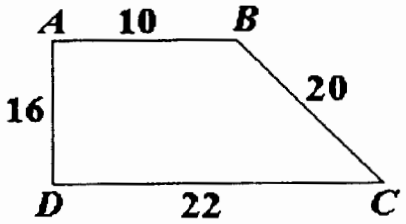


3. In ΔABC, the measure of $\angle A$ is $\left(c + \frac{d}{3}\right)^\circ$ and the measure of $\angle B$ is $\left(2b + \frac{c}{4}\right)^\circ$.

Express the degree measure of the supplement of $\angle C$ as a reduced fraction.

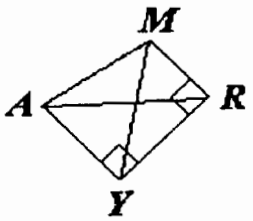
4. Let x be the overall average rate, in mph, for a two-way trip in which one way was traveled at 40 mph and the other way was traveled at 45 mph. (Assume the same route is traveled in both directions.) Let y be the perimeter of a rectangle whose sides are integers and whose diagonal is 15. Find the maximum of x and y.

5. In trapezoid $ABCD$, shown below, $AB \parallel DC$, $AD \perp DC$, $AB = 10$, $BC = 20$, $CD = 22$, and $DA = 16$. If the trapezoid is revolved about the segment DC, what is the volume of the resulting solid?



6. If the radius of the circle $x^2 + y^2 + 18x - 12y + 17 = 0$ is r and p is the solution to $\log_2(3p) + \log_2(2p - 1) = \log_2 9$, find p/r.

7. In the following diagram, $AY \perp YR$, $AM \perp MR$, and $AY = YR$. If $AR = 12$ and the measure of $\angle MAR$ is 30°, find the length of MY. Give your answer correct to four significant digits.



8. Let n be defined by $2^4 4^2 = n^8$. Let r be the remainder when $x^3 - 31x + 30$ is divided by $x - 1$. What is the value of the product nr ?

9. If f and g are defined by $f(x) = 3x + 2$ and $g(x) = 3x^2$, respectively, find $g\left(f^{-1}\left(\frac{3}{2}\right)\right)$.

10. Find the area of the triangle whose vertices are (2, 1), (4, 5), and (-3, 2).