NMAθ 2002
ALPHA State Bowl

Round # _____

A = _____

B = _____

C = _____

Final answer:

CODE: _____

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Final answer:

CODE: _____

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Round 1

Part 1
A sequence is defined as $a_1 = 10$ and

$$a_n = \begin{cases} \frac{a_{n-1}}{2} & if \ \ a_{n-1} \ is \ even \\ 3(a_{n-1})+1 & if \ a_{n-1} \ \ is \ \ odd \end{cases},$$

find the 10th term.

A = the 10th term

Part 2
Find the sum of the infinite geometric series: $6 + \sqrt{6} + 1 + \frac{1}{\sqrt{6}} + \ldots$

B = denominator after rationalizing

Part 3
Find the product of x and y if $y = 4\log_4 x + 1$ and $y - 3 = 2\log_4 x.$

C = the product

Final answer = $|A - B - C|$

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Round 2

Part 1
P(x) is an odd function such that P(1) = 2, P(3) = 5 and P(-5) = -1. What is the value of P(P(P(- 3))) ?

A = answer

Part 2
θ is in Quadrant II with $\cos\theta = -\frac{3}{5}$,
calculate $(\cos\theta + \sin\theta)^2$

B = answer

Part 3
Find the value for k that will make $x-6$ a factor of the polynomial:
$x^3 - kx^2 - 21x - 18$

C = k

Final: $B^{(A+C)}$

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Round 3

Part 1
For $1 < x < y < x+y$, let $S = \{1, x, y, x+y\}$. What is the difference between the mean and the median of S?

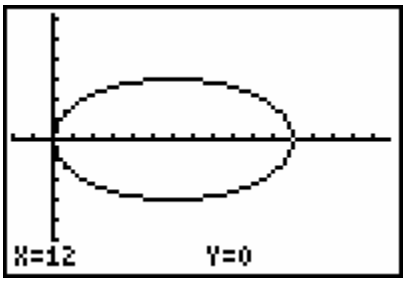
A = the difference

Part 2

A punch bowl is in the shape of a hemisphere with diameter 36 cm. How deep is the punch in the bowl if the punch begins to pour when the bowl is tilted 30° to the horizontal?

B = the depth of the punch

Part 3
Write the equation of the ellipse.



C = first coordinate of the foci located on the right side of the center

Final: $(2A)(B) + C$

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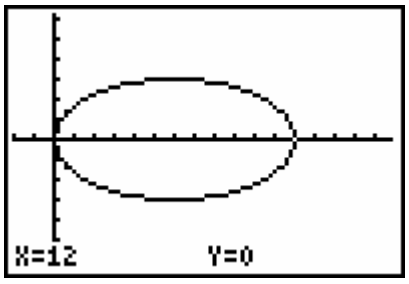

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Round 4

Part 1
Find the solutions for the system:
$x^2 + y^2 - 6x + 8y - 9 = 0$
$x^2 + y^2 - 6x - 4y - 21 = 0$

A = the product of all the coordinates

Part 2
Given $f(x) = x^3 + Ax^2 + Bx - 3,$
$f(1) = 4, f(-1) = -6$

B = $A + B$

Part 3
Find x, where $0 < x < 90^\circ$:
$81^{\cos^2 x}\, 9^{\cos x} = 729^{\frac{1}{3}}\, 3^{2\cos x}$

C = x

Final: $A + B + C$

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C = x

Final: $A + B + C$

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Round 5

Part 1
Solve for x:

$$\begin{vmatrix} 4 & x^2 & x \\ 3 & 1 & 0 \\ -1 & -2 & 3 \end{vmatrix} = 8$$

A = integral answer

Part 2
Find the y-intercept of the graph of: $y = \frac{3}{4}\sin\left(2x - \frac{3\pi}{2}\right)$. Let β be a QII angle and $\cot\beta =$ the value of the y-intercept you found in the first part of the problem.

B = $2\sin\beta + 3\cos\beta$

Part 3
If the coefficient of the 5th and 6th terms in the expansion of $(x-y)^n$ are equal, find the 3rd term.

C = the coefficient divided by the degree of the term

Final: $\text{int}\left(C^A + B\right)$

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Round 6

Part 1

$2\log_2(x-2) - \log_2(x+1) =$
$\log_2 0.4 + \log_2 10$

A = x

Part 2
What is the least value of y that satisfies the following inequality?
$|4+x| + |5+y| \le 100$

B = the least value of y

Part 3
Three friends each have a red, a white, a yellow, a blue and a green T-shirt. If each of them randomly chooses a T-shirt to wear, what is the probability that they all choose different colors?

C = probability

Final: $C(B)(A^{-1})$

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Round 6

Part 1

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Round 7

Part 1
What number(s) have the property that its reciprocal is one less than the number?

A = sum of the answers

Part 2
In the sequence,
..., a, b, c, d, 0, 1, 1, 2, 3, 5, ...
each term is the sum of the two preceding terms. What is the value of a?

B = a

Part 3
The fraction below contains the nine digits 1, 2, 3, 4, 5, 6, 7, 8, and 9. Find the appropriate positions of the remaining digits.

$$\frac{\bullet\ 6\ \bullet\ \bullet}{\bullet\ \bullet\ 3\ \bullet\ \bullet} = \frac{1}{2}$$

C = the digit in the units position of the denominator

Final: $|A + B| + C$

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Final: $|A + B| + C$

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Round 8

Part 1
The angle of elevation to a jet is 15° from a ship 50 miles off shore. What is the height of the jet as it passes directly over the shoreline?

A = height of the jet

Part 2
a + b + c + d = 72, where a, b, c and d are four distinct numbers. If five is added to a, subtracted from b, multiplied by c and divided into d, the value of each resulting expression is the same. Find the four distinct numbers.

B = sum of the digits of the four distinct numbers

Part 3
How many integers x in
{1, 2, 3, ..., 99, 100} are there such that $x^2 + x^3$ is the square of an integer?

C = answer

Final: $A\left(\frac{B}{C}\right)$

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Round 9

Part 1
During a recent span of time, eleven days had some rain. A morning rain was always followed by a clear afternoon, and an afternoon rain was always preceded by a clear morning. In all, nine mornings and twelve afternoons were clear.

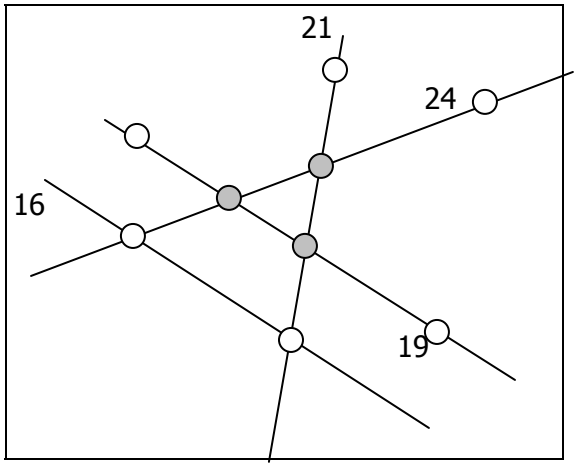
A = number of days with no rain at all

Part 2
Given all X's are Y's, but only some Y's are Z's. Which of the following statements are true?
(1) No X's can be Z's
(2) If something is not a Y, then it is not an X.
(3) If something is a Z, then it is not an X.

B = the number (or the sum of the numbers) of the true statement(s)

Part 3
Put a different number from 1 to 9 in each of the boxes in the diagram so that the sum along each line is the number shown.

C = answer



Final: $6A - 4B + C$

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During a recent span of time, eleven days had some rain. A morning rain was always followed by a clear afternoon, and an afternoon rain was always preceded by a clear morning. In all, nine mornings and twelve afternoons were clear.

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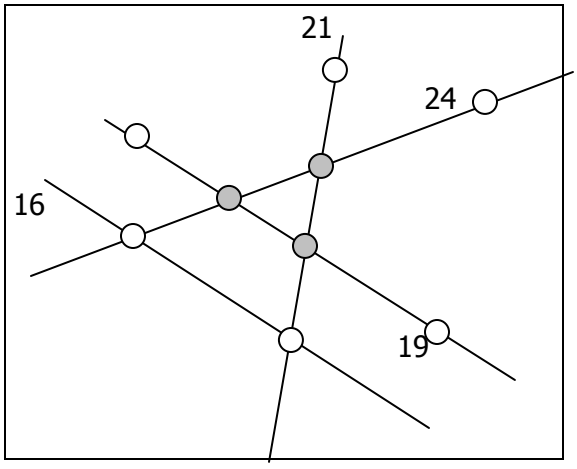

Final: $6A - 4B + C$

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Round 10

Part 1

If $\tan A = \frac{2xy}{x^2 - y^2}$ where x>y>0, and

$0 < A < \frac{\pi}{2}$, find an expression for $\sin A$

A = writing the fraction as a single polynomial, find the sum of the coefficients

Part 2

First find the roots of
$x^3 + 4x^2 - 7x - 10 = 0$ then use that answer to determine the roots of
$(x-3)^3 + 4(x-3)^2 - 7(x-3) - 10 = 0$

B = the smallest three digit number, disregarding any signs

Part 3
An urn contains 100 black and 100 white marbles. Repeatedly, 3 marbles are removed and replaced as follows:

Marbles removed	replaced with
3 black	1 black
2 black, 1 white	1 black, 1 white
1 black, 2 white	2 white
3 white	1 black, 1 white

C = the probability that all are white when fewer than 3 marbles remain

Final: $\frac{\sqrt{B}}{(A+C)}$

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