Mu Alpha Theta National Convention 2003
Alpha Applications – Alpha Level

For all questions, answer choice "E" – NOTA, means none of the above answer is correct.

1) If you roll a 1-meter diameter wheel forward 30cm over level ground, through what angle (in radians) will the wheel turn?

A) $\frac{3}{5}$ B) $\frac{3\pi}{5}$ C) $\frac{3}{10}$ D) $\frac{3\pi}{10}$ E) NOTA

2) What is the volume of the parallelepiped spanned by the vectors: $2i + j - k$, $5i - 3k$, and $i - 2j + k$?

A) 5 B) 10 C) 20 D) 60 E) NOTA

3) Find the smaller angle between the hands of a clock when the time is 11:43 a.m., and convert this angle to radian measure. What is the $5^{th}$ digit after the decimal place in the decimal expansion of this radian measure.

A) 2 B) 3 C) 6 D) 8 E) NOTA

4) Given the sequence 2, 6, 10, 14, … find the $50^{th}$ term in this sequence.

A) 196 B) 198 C) 200 D) 202 E) NOTA

5) Given the formula: $e^{-d} = \tan(\Theta/2)$, where $d$ is the hyperbolic distance from a point $p$ to a hyperbolic straight line and $\Theta$ is the angle of parallelism of $p$ with respect to this line, what is the distance $d$ such that the angle of parallelism is $\frac{\pi}{4}$?

A) $\ln(\sqrt{2}-1)$ B) $\ln(\sqrt{2}+1)$ C) $\ln(\sqrt{3}-1)$ D) $\ln(\sqrt{3}+1)$ E) NOTA

6) Determine the value of $(1+i)^{2001} + (1-i)^{2001}$.

A) $2^{999}$ B) $2^{1000}$ C) $2^{1001}$ D) $2^{1002}$ E) NOTA

7) Suppose that $z$ and $w$ are nonzero complex numbers such that $|z| = |w| = r > 0$. Then which of the following is always true?

A) $z$ and $w$ are parallel
B) $z$ and $w$ are orthogonal
C) $z$ and $w$ have the same direction
D) $z \bullet w \neq 0$
E) NOTA

8) Find the smallest positive integers $m$ and $n$ which satisfy the following equation: $(1+i\sqrt{3})^m = (1-i)^n$.

A) $\begin{matrix} m=2 \\ n=4 \end{matrix}$ B) $\begin{matrix} m=6 \\ n=12 \end{matrix}$ C) $\begin{matrix} m=16 \\ n=32 \end{matrix}$ D) $\begin{matrix} m=24 \\ n=48 \end{matrix}$ E) NOTA

9) Define $\Theta$ to be the exterior angle of a regular n-gon. Compute: $\sum_{k=0}^{n-1} \cos(k\Theta)$.

A) 0 B) 1 C) $\pi$ D) $2\pi$ E) NOTA

10) An airliner is flying in a circular holding pattern with a radius of 12 miles over New York City's Laguardia Airport. Find the central angle (in radians) that the airliner will have flown through by flying 12 miles along this circular path.

A) $\pi$ B) 1 C) $\frac{\pi}{2}$ D) $\frac{\pi}{6}$ E) NOTA

11) Assuming that the earth's radius is approximately 4000 miles and that the earth makes one full rotation ( $2\pi$ radians) every 24 hours, about how fast are you moving if you "stand still" on the equator?

A) 10 mph B) 150 mph C) 1047 mph D) 2447 mph E) NOTA

12) Suppose there are 12 men and 7 women auditioning for television's new reality series and that the producers of the series are looking for six teams of two competitors to compete for the ultimate prize of ten million dollars. Each team must be made up of one male and one female. How many ways can the producers choose their cast for their new series?

A) 6468 B) 38808 C) 4656960 D) 9313920 E) NOTA

13) Let $f(x) = 4\sin(3x-16)+2$. Define A = amplitude, B = period, C = phase shift, D= vertical shift, and E = the number of roots of $f(x)$ on the interval $0 < x \le 2\pi$. Compute $\frac{A+B+C+D-E}{E^2}$. Round your answer to the nearest hundredth place.

A) .21 B) .54 C) 1.22 D) 7.43 E) NOTA

14) Consider the tetrahedron with vertices (1, 2, 1), (2, 3, -1), (1, 1, 1) and (1, -4, -2). Let $A =$ the volume of this tetrahedron, $B =$ the perimeter of this tetrahedron. Find AB. (round to the nearest whole number)

A) 7 B) 9 C) 26 D) 52 E) NOTA

15) Todd invests $1000 at 15% compounded quarterly. Jennifer invests $1000 at 15% compounded continuously. After 10 years, how much more is Jennifer's investment worth than Todd's investment?

A) $101.23 B) $121.31 C) $143.21 D) $161.31 E) NOTA

16) A radioactive substance has a half-life of 30 years. If at time zero you have 200 grams of this substance, how many grams will remain after 75 years? (round to the nearest hundredth)

A) 20.26 B) 29.35 C) 35.36 D) 41.47 E) NOTA

17) Determine the $11^{th}$ partial sum of the sequence: 1, 1, 2, 3, 5, …

A) 89 B) 123 C) 143 D) 232 E) NOTA

18) You want to make an $70^\circ$ angle by marking an arc on the perimeter of a 10 inch diameter circle and drawing lines from the ends of the arc to the center of the circle. To the nearest tenth, how long should the arc be?

A) 3.2 in. B) 4.6 in. C) 6.1 in. D) 7.5 in. E) NOTA

19) Consider the following matrix $M = \begin{bmatrix} 1 & 4 & 7 \\ 2 & -3 & 6 \\ 4 & 0 & 7 \end{bmatrix}$. What is the element in the $2^{nd}$ row , $3^{rd}$ column of the following computation:

(minor of 2)(cofactor matrix of $M$ )(adjoint matrix of $M$ )?

A) 19180 B) –27916 C) 15960 D) –44912 E) NOTA

20) Determine the radius of the unique circle which passes through the points (-1, 5), (5, 5), and (7, 1).

A) 4 B) 5 C) 6 D) 7 E) NOTA

21) If the radius of a right cylinder is increased by 20%, while the height is decreased by 10%, by what percent does the volume of the cylinder increase?

A) 10% B) 29.6% C) 30% D) 32.4% E) NOTA

22) Given that $\cos(a)+\cos(b)=2\cos(\frac{7}{2})\cos(\frac{3}{2})$, where $a,b\in\mathbb{Z}$ and $a\geq b$, find $a$.

A) 2 B) 3 C) 4 D) 5 E) NOTA

23) A projectile fired from the ground follows a parabolic path. The maximum height of the projectile is 500 ft. and it strikes the ground 1000 ft. from the point at which it was fired. When was the projectile 150 ft. above ground, and falling towards the ground?

A) 200ft. B) 300ft. C) $50\sqrt{70}$ D) $500+50\sqrt{70}$ E) NOTA

24) Find the cosine of the smaller angle between the asymptotes of the following hyperbola: $x^2+2x-4y^2=3$.

A) $\frac{3}{5}$ B) $\frac{4}{5}$ C) 1 D) 0 E) NOTA

25) Using Descartes' Rule of Signs, what is the maximum number of negative roots of the polynomial: $P(x)=x^5-2x^4-9x^3+8x^2-22x+24$.

A) 1 B) 2 C) 3 D) 4 E) NOTA

26) Find the area of the region containing the points satisfying $|x-1|+|y+2|\geq 2$ and $(x-1)^2+(y+2)^2\leq 4$.

A) $2\pi-8$ B) $8-4\pi$ C) $8+4\pi$ D) $8\pi-8$ E) NOTA

27) Suppose this test is scored in the following manner: 4 points for a correct answer, -1 points for an incorrect answer, and 0 points for an unanswered question. How many different combinations of correct, incorrect, and unanswered questions are there in order to score a total of 20 points?

A) 2 B) 4 C) 6 D) 8 E) NOTA

28) Let $A=\sqrt{5+\sqrt{5+\sqrt{5+...}}}$ and $B=\sqrt{5-\sqrt{5-\sqrt{5-...}}}$. Find $A-B$.

A) 1 B) 2 C) $\sqrt{5}$ D) $1-\sqrt{5}$ E) NOTA

29) A box contains 45 marbles, 20 of which are red, 15 green, and 10 blue. If six marbles are picked from the box at random, what is the probability that all six marbles are the same color?

A) $\frac{210}{8,145,060}$ B) $\frac{5005}{8,145,060}$ C) $\frac{43,365}{8,145,060}$ D) $\frac{43,975}{8,145,060}$ E) NOTA

30) Find the polar equation of the ellipse that has vertices at the points with polar coordinates (1, 0) and (3, $\pi$ ).

A) $r=\frac{3}{2+\cos\phi}$ B) $r=\frac{2}{3+\cos\phi}$ C) $r=\frac{1}{2+\cos\phi}$ D) $r=\frac{3}{3+\cos\phi}$ E) NOTA