Alpha Equations and Inequalities
Mu Alpha Theta National Convention 2003
Note: The answer choice E (NOTA) stands for "none of the above"

1) Determine the amplitude of the function h(s)= 5sin(6s+7) + 9cos(6s+7).

(A) $\sqrt{6}$ (B) $\sqrt{14}$ (C) $\sqrt{106}$ (D) $\sqrt{7}$ (E) NOTA

2) Given the function f(x) with the property f(x+k) = f(x)$\bullet$f(k) for f(k)>0, what is the y-intercept of f(x)?

(A) 0 (B) 1 (C) 3 (D) 3.5 (E) NOTA

3) What is the sum of the complex solutions to the equation $x^3 - 5x^2 + 9x - 5 = 0$?

(A) 1 (B) 4 (C) 5 (D) 7 (E) NOTA

4) D = The maximum value of $\dfrac{4}{3+\cos\theta}$ where $0 \le \theta \le 2\pi$.

E = The minimum value of $\dfrac{82 + 9x^2 + 9x}{3(3+x)}$.

Find <D,E>•<E,D>. (NOTE: • stands for dot product)

(A) 4 (B) 6 (C) 8 (D) 12 (E) NOTA

5) Find the point of symmetry for the equation $x^3 - 4x^2 + 7x - 4 = y$.

(A) $(\frac{2}{3}, \frac{-22}{27})$ (B) $(\frac{4}{3}, \frac{16}{27})$ (C) (2,2) (D) (0,–4) (E) NOTA

6) The We Say So Corporation sells political propaganda materials such as bumper stickers, pins and advertisement signs with sale prices per gross (1 gross = 144 units). 288 bumper stickers, 576 pins and 144 advertisement signs cost $85. 576 bumper stickers, 288 pins and 288 advertisement signs cost $80. 288 pins, 288 bumper stickers and 720 advertisement signs cost $75. How much would it cost if you wanted to buy 2 gross of each product?

(A) 60 (B) 70 (C) 80 (D) 105 (E) NOTA

7) Given that $x > e$, what is the range of the function Arctan(ln(x))?

(A) $(0, \pi)$ (B) $(0, 2\pi)$ (C) $(-\frac{\pi}{2}, \frac{\pi}{2})$ (D) $(0, \frac{\pi}{2})$ (E) NOTA

8) $\lim_{b\to\infty} \sum_{n=1}^{b} \frac{1}{(n+2)(n+3)} = x$. What does x equal?

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{6}$ (D) $\frac{1}{12}$ (E) NOTA

9) How many possible solutions are there to the equation $|\sec x| = \left|\csc \frac{\pi}{3}\right|$ within the domain $0 \le x \le 2\pi$?

(A) 2 (B) 3 (C) 4 (D) 8 (E) NOTA

10) $(x^2 - 11x - 11)^{(x^2+8x+15)} = 1$. Solve for x.

(A) x = -12,-3,-1,5 (B) x = -3,-1,0,12
(C) x = 0,1,3,5,12 (D) x = -5,-3,-1,12 (E) NOTA

11) Jason bought Boggle-Juice® for 11 dollars a gallon. Sasha, who is an expert haggler, was able to buy the same Boggle-Juice® for only 8 dollars a gallon. If the total amount of money they spent on Boggle-Juice® was 345 dollars, what is the sum of all distinct possible volumes of Boggle-Juice® they could have purchased together? (NOTE: You may only purchase non-negative integer amounts of gallons)

(A) 42 (B) 81 (C) 97 (D) 150 (E) NOTA

12) Which of the following statements are true about inverse functions?

I . Points of intersection of $f(x)$ and $f^{-1}(x)$ are in the form (a,-a), for some integer a
II. Points of intersection of $f(x)$ and $f^{-1}(x)$ are in the form (a, b) for some integers a and b
III. e^x and ln(x) are inverse functions

(A) II and III (B) I and III (C) Only II (D) Only III
(E) NOTA

13) What is the sum of all integer values of m that make $\frac{3m+25}{2m-7}$ a positive integer?

(A) 63 (B) 70 (C) 87 (D) 112 (E) NOTA

14) Suppose there are two trains coming towards each other on the same track, Train A going 85 m/s and Train B going 115 m/s. The distance between the two trains at t=0 is 4000m. Now, suppose there is a bumble bee on the tip of Train A who, at t=0, bolts towards Train B. When it reaches Train B, it immediately turns around and goes back towards train A. It continues to go back and forth between the trains until they crash. If the trains will definitely collide and the Bee's speed is 456 m/s, what is the total distance traveled by the Bee from t=0 until the two trains crash?

(A) 4560 m (B) 7752 m (C) 9120 m (D) 10488 m (E) NOTA

15) If $f(x)=4x^3-10x+\cos(x)$, $g(x)=x^3-10x+4\sin(x)$, and $h(x)=7x^5-\cos(x)$ then which arithmetic combination of functions yields an odd function?

(A) f(x) – h(x) + g(x) (B) f(x) + h(x) + g(x)
(C) g(x) + h(x) – f(x) (D) 2f(x) – h(x) (E) NOTA

16) What is the area of the region enclosed by the inequalities T_1 < x, T_5 > x, -T_4 < y, and y < T_6, given that T_n is the nth triangular number?

(A) 1511 (B) 434 (C) 465 (D) 165 (E) NOTA

17) Solve for m: $m+2 < -2m + 13 < 4m - 6$.

(A) m < $\frac{11}{5}$ (B) m > $\frac{-7}{6}$
(C) $\frac{-7}{6}$ < m < $\frac{11}{5}$ (D) $\frac{-11}{5}$ < m < $\frac{7}{6}$ (E) NOTA

18) Hahvahd University accepts an average of A students per year. Fale University accepts and average of B students per year. Given that A and B are both real numbers and B > A > 1, since Hahvahd is much more selective than Fale, which of the following always has the greatest value?

(A) The Arithmetic Mean of A and B
(B) The Geometric Mean of A and B
(C) The Harmonic Mean of A and B
(D) The Root Mean Square of A and B (E) NOTA

19) What is the shortest distance between y = 4x+9 and y=4x+11?

(A) $\frac{3\sqrt{7}}{7}$ (B) $\frac{2\sqrt{15}}{15}$ (C) 3 (D) $\frac{2\sqrt{17}}{17}$ (E) NOTA

20) Billie Joe, Mike and Tre each start a new back account at the We-use-your-money Bank. Billie Joe deposits \$1000 at 6% interest, compounded quarterly for 10 years, Mike deposits \$990 at 6.5% interest compounded semi-annually for 10 years and Tre deposits \$980 at 6.1% interest compounded continuously for 10 years, What will the sum of all their bank accounts be at the end of 10 years, assuming that Mike spends 19% of his final 10-year balance on Post-it notes? (Round your answer to the nearest cent).

(A) \$5137.90 (B) \$5137.91 (C) \$5494.52 (D) \$5494.53 (E) NOTA

21) If $\frac{x+y}{xy} = 7$ and $x^2 + y^2 = \frac{37}{36}$, what does $(x + y)^3$ equal given that both x and y are positive?

(A) $\frac{27}{8}$ (B) $\frac{1331}{729}$ (C) $\frac{343}{216}$ (D) $\frac{512}{27}$ (E) NOTA

22) If f(x) = 2x + 3, and f(f(f(f(f(x))))) = ax + b what is a+b?

(A) 125 (B) 173 (C) 106 (D) 77 (E) NOTA

23) If $x + \frac{1}{x} = 3$, then what does $x^3 + \frac{1}{x^3}$ equal, given the x is complex number?

(A) 16 (B) 17 (C) 18 (D) $\frac{244}{24}$ (E) NOTA

24) You are given the graphs y=sin(2x)+5 and y=cos(2x)-2. Which of the following shifts make the sine graph perfectly coincidental with the cosine graph?
(Positive values denote a shift up or right and negative values denote a shift down or left, depending on which axis)

(A) X-shift: π Y-shift: 7
(B) X-shift: 0 Y-shift: -7
(C) X-shift: $-\pi/2$ Y-shift: -7
(D) X-shift: $-\pi/3$ Y-shift: -7
(E) NOTA

25) Find the area that satisfies the equation $\lfloor x \rfloor^2 + \lfloor y \rfloor^2 = 400$. (NOTE: $\lfloor x \rfloor$ is defined as the greatest integer less than or equal to x)

(A) 0 (B) 12 (C) 16 (D) 24 (E) NOTA

26) What is the area of the region satisfying the equations y $\geq$ |x| and 3y-x $\leq$ 8?

(A) $\sqrt{10}$ (B) $2\sqrt{10}$ (C) 16 (D) 4 (E) NOTA

27) $\log_a b = 4$. $\log b + \log a = 5$. How many positive integral factors does (b-a) have?

(A) 24 (B) 30 (C) 32 (D) 36 (E) NOTA

28) x is directly proportional to z^3 and inversely proportional to y^2. When z = 2 and y = 5, x = 200. What can y equal when x = 35 and z = 7?

(A) $30\sqrt{5}$ (B) $17\sqrt{10}$ (C) $35\sqrt{5}$ (D) $29\sqrt{10}$ (E) NOTA

29) $\frac{1}{\log_2 100!} + \frac{1}{\log_3 100!} + \frac{1}{\log_4 100!} + \ldots + \frac{1}{\log_{99} 100!} + \frac{1}{\log_{100} 100!} = m$. What is m^3?

(A) 1 (B) 8 (C) 27 (D) 125 (E) NOTA

30) Find the solution interval for $12x^2 - 11x - 15 \geq 0$.

(A) $(-\frac{3}{4}, \frac{5}{3}]$ (B) $[-\frac{3}{4}, \infty)$

(C) $(-\infty, -\frac{3}{4}]$ U $[\frac{5}{3}, \infty)$ (D) $[-\frac{3}{4}, \frac{5}{3})$ (E) NOTA