Mu Alpha Theta National Convention 2003
Alpha Division
Page 1

NOTE: NOTA means "None of the above."

1. The remainder when $x^{2003} + 2002$ is divided by $x - 1$ is
   a. -1 b. 2002 c. 2003 d. 2004 e. NOTA

2. Tan $\pi/4$ =
   a. 1 b. $\sqrt{3}/3$ c. $\sqrt{3}$ d. 5 e. NOTA

3. $\cos^2\theta - \sin^2\theta$ =
   a. –1 b. 1 c. $\sin 2\theta$ d. $\cos 2\theta$ e. NOTA

4. The distance between the two polar points $(2, 7\pi/12)$ and $(1, -\pi/12)$ is
   a. $\sqrt{5}$ b. $\sqrt{7}$ c. $\sqrt{3}$ d. $\frac{3\sqrt{3}}{2}$ e. NOTA

5. The equation of one of the asymptotes of the hyperbola defined by $9x^2 - 16y^2 - 18x - 64y - 199 = 0$ is:
   a. $3x + 4y = -5$ b. $3x - 4y = -5$ c. $4x - 3y = 7$
   d. $4x + 3y = -2$ e. NOTA

$$r = \frac{5}{2 - 3\sin\theta}$$

6. The graph of the polar equation represents a(n):
   a. parabola b. cardioid c. ellipse d. hyperbola e. NOTA

7. $\sum_{n=4}^{\infty} \frac{5}{(n^2 - n - 6)} =$
   a. $\frac{137}{60}$ b. $\frac{55}{24}$ c. $\frac{34}{15}$ d. $\frac{103}{45}$ e. NOTA

8. Given $f(x) = 6x^9 - 3x^5 - 4x^3 + 7$ and $f(a) = -8$, then $f(-a)$ =
   a. 16 b. 8 c. 1 d. 22 e. NOTA

9. If $\log 216 = a$ and $\log 625 = b$, then $\log (1/12)$ =
   a. $\frac{4b + 3a - 24}{12}$ b. $\frac{4a - 3b - 12}{12}$ c. $\frac{12}{3b - 4a - 12}$ d. $\frac{3b - 4a - 12}{12}$ e. NOTA

10. Find all of the asymptotes of the given function:

$$f(x) = \frac{4x^3 - 108}{5x^3 - 2x^2 - 45x + 18}$$

a. $y = 0.8, x = -3, x = 3, x = 0.4$
b. $y = 0.8, x = 3, x = 0.4$
c. $y = 0.8, x = -3, x = 0.4$
d. $x = -3, x = 3, x = 0.4$
e. NOTA

11. If $8\sin^2 x - 8\cos^2 x = -6$, and $0.5\pi < x < \pi$, then $\sin 2x =$

a. $-\frac{4}{5}$ b. $-\frac{3}{5}$ c. $\frac{\sqrt{7}}{4}$ d. $\frac{\sqrt{7}}{3}$ e. NOTA

12. The shortest distance between the point (-3, 5, 2) and the plane $2x - 3y + 2z = 4$ is

a. $\frac{21\sqrt{17}}{17}$ b. $\frac{29\sqrt{17}}{17}$ c. $\frac{21\sqrt{38}}{38}$ d. $\frac{29\sqrt{38}}{29}$ e. NOTA

13. If $x - y = 28$ when $x^3 - y^3 = 1456$, then the value of xy is
a. 278 b. 52 c. - 20 d. - 48 e. NOTA

14. Find the eccentricity of the graph in the xy-plane of

$$x = 8\tan t - 3$$
$$y = 4\sec t + 2$$

a. $\frac{\sqrt{5}}{2}$ b. $\frac{2\sqrt{5}}{5}$ c. $\frac{2\sqrt{3}}{3}$ d. $\sqrt{5}$ e. NOTA

15. If $f(x) = \frac{3x+2}{2x-3}$, find $f^{-1}\left(\frac{3}{4}\right)$.

a. $-\frac{17}{6}$ b. $-\frac{6}{17}$ c. $\frac{6}{17}$ d. $\frac{17}{6}$ e. NOTA

16. Find the maximum value of the function $f(x) = 3\sin(2x) + 4\cos(2x) + 6$

a. 11 b. 12 c. 13 d. 14 e. NOTA

17. $\dfrac{\sin 50^\circ}{6\sin^2 20^\circ - 3} =$

a. $\tan 70^\circ$ b. 0.5 c. –1/4 d. – 1/3 e. NOTA

18. One out of every three nutritionists recommend Brand X. If five nutritionists are asked their opinions on Brand X, what is the probability that at least four out of the five will recommend Brand X?

a. $\dfrac{10}{243}$ b. $\dfrac{11}{243}$ c. $\dfrac{4}{81}$ d. $\dfrac{5}{81}$ e. NOTA

19. $(-\sqrt{3}+i)^{10} =$

a. $512 + 512\sqrt{3}i$ b. $512\sqrt{3} + 512i$ c. $512 - 512\sqrt{3}i$ d. $-512\sqrt{3} + 512i$ e.NOTA

20. Find the sum of the roots of the function $f(x) = e^{3x} - 4e^{2x} - 9e^{x} + 36$.

a. 4 b. – 4 c. $2\ln 6$ d. $\ln 12$ e. NOTA

21. Find the determinant value:

$$\begin{vmatrix} \log_7 125 & \log_{16} 27 \\ \log_9 8 & \log_5 49 \end{vmatrix}$$

a. 5 b. $\log_4 35$ c. $\dfrac{39}{8}$ d. 6 e. NOTA

22. The sum of the coefficients of the expansion of $(3x - y)^5$

a. 128 b. 64 c. 32 d. 30 NOTA

23. Independently of each other, two numbers are selected at random from among the positive real numbers less than three, with all numbers equally likely to be selected. What is the probability that the sum of the squares of the two selected numbers is less than nine?

a. $\frac{\sqrt{2}}{2}$ b. $\frac{\sqrt{3}}{3}$ c. $\frac{\pi}{4}$ d. $\frac{\pi}{3}$ e. NOTA

24. Find the exact value of $g(\pi/48)$ when $g(x) = \log_2(\sin x) + \log_2(\cos(x)) + \log_2(\cos(2x)) + \log_2(\cos(4x))$
a. -8 b. -4 c. 4 d. 8 e. NOTA

25. The number of positive even factors of 3000 is
a. 32 b. 28 c. 26 d. 23 e. NOTA

26. The angle of intersection of the two lines $7x - y = 5$ and $3x - 4y = 7$ is
a. $\pi/6$ b. $\pi/5$ c. $\pi/4$ d. $\pi/3$ e. NOTA

27. Find the exact value of $\sin(5\pi/18)\cos(5\pi/9) + \sin(5\pi/9)\cos(5\pi/18) =$
a. $-\frac{\sqrt{3}}{2}$ b. $\frac{\sqrt{3}}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ NOTA

28. Simplify: $\frac{(\cos x + 1 + \sin x)(\sin x - 1 + \cos x)}{\cos^4 x + 1 - \sin^4 x}$

a. $\sin 2x$ b. $\tan x$ c. $\cos^2 x$ d. 1 e. NOTA

29. The roots of $x^3 + 8x^2 + 4x + 2 = 0$ are *r*, *s*, and *t*. For what ordered triple of integers (*a*, *b*, *c*) are *3r*, *3s*, and *3t* the roots of $x^3 + ax^2 + bx + c = 0$?
a. (24, 12, 6) b. (24, 36, 54) c. (8, 12, 18) d. (32/3, 16/3, 16/9) e. NOTA

30. Find the volume of the parallelepiped determined by the vertices (-2, 4, 1), (3, 5,2), (1, -2,4), and (-4, 3, -3).

a. 126 b. 127 c. 128 d. 129 e. NOTA