This test consists of five relays of six questions each. "TAFTPQITR" stands for "the answer from the previous question in the relay", so if question 3 in a relay references TAFTPQITR, that is the answer from question 2 in that relay.

All answers on this test are integers.

Relay 1

1. Find the magnitude of the vector $\langle -6,-7,6 \rangle$.

2. Let $T =$ TAFTPQITR. 2^n is part of the prime factorization of $(T+1)^4$. Find the largest integral value of n.

3. Let $T =$ TAFTPQITR. Find the number of degrees in the sum of the interior angles of a convex T-gon.

4. Let $T =$ TAFTPQITR. How many positive integral divisors does T have?

5. Let $T =$ TAFTPQITR. Let $N =$ the sum of the digits of T. Find the real Nth root of T.

6. Let $T =$ TAFTPQITR. Find the value of $A(T,T)$, where $A(m,n)$ is the Ackermann function defined by $A(m,n) = \begin{cases} n+1, \text{ if } m=0 \\ A(m-1,1), \text{ if } m>0 \text{ and } n=0 \\ A(m-1,A(m,n-1)), \text{ if } m>0 \text{ and } n>0 \end{cases}$.

Relay 2

1. If $\cos\theta = -127/128$, where $\pi/2 < \theta < \pi$, find the value of $\sec(\theta/2)$.

2. Let $T =$ TAFTPQITR. The point $(T,-63)$ is what distance away from the origin?

3. Let $T =$ TAFTPQITR. A fair quarter is flipped T times. The probability of flipping at most two heads can be written as A/B, where A and B are relatively prime positive integers. Find the value of A.

4. Let $T =$ TAFTPQITR. Find the sum of the entries of the product of T copies of the matrix $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$.

5. Let $T =$ the sum of the digits of TAFTPQITR. There are two two-digit positive integers that are equal to three times the product of their digits. One is T. What is the other?

6. Let $T =$ TAFTPQITR. Let X be a set with T distinct elements. How many functions exist whose domain is $\{0,1\}$ and range is a subset of X?

Relay 3

1. Find the remainder when $x^5 + 12x^4 + 3x^3 - 20x^2 + 13x + 5$ is divided by $x + 2$.

2. Let $T =$ TAFTPQITR. The graph of $y = ax^2 + bx + c$ goes through the points $(2,T)$, $(3,57)$, and $(-1,29)$. Find the value of c.

3. Let $T =$ TAFTPQITR. When $\dfrac{12x^2 + 27x + T}{x^3 - x^2 + 4x - 4}$ is decomposed into partial fractions, what is the sum of the coefficients of the two numerators?

4. Let $T =$ TAFTPQITR. The variable X is normally distributed with mean 12 and standard deviation 3. A score of T in this distribution lies how many standard deviations away from the mean?

5. Let $T =$ TAFTPQITR. Find the value of a_T in the sequence defined by $a_n = 3a_{n-1} - 2a_{n-2}$ for $n \geq 3$, with $a_1 = 2$ and $a_2 = 3$.

6. Let $T =$ TAFTPQITR. For how many positive integers a is $\log_a(T-1)$ an integer?

Relay 4

1. Find the value of $\left|(3-2i)^4\right|$.

2. Let $T =$ TAFTPQITR. Find the value of a_T in the sequence defined by $a_n =$ the sum of the squares of the digits of a_{n-1} for $n \geq 2$, with $a_1 = 2011$.

3. Let $T=$TAFTPQITR. A right circular cone with altitude T and base radius 3 has volume $n\pi$. Find the value of n.

4. Let $T=$TAFTPQITR. For positive integer n, let $P(n)=$the product of the digits of n. Find the value of $P(T^2)$.

5. Let $T=$TAFTPQITR. If A and B are positive integers, with $A<B$, such that $\frac{1}{A}+\frac{1}{B}=\frac{1}{T}$, and $A+B$ is as small as possible, find the value of A.

6. Let $T=$TAFTPQITR. T° is the measure of angle θ. Three times the complement of θ minus the supplement of θ is equal to how many degrees?

Relay 5

1. Of all Pythagorean triples that include the number 25, let M be the largest number that appears in any of the triples, and let m be the smallest number that appears in any of the triples. Find the value of $M-m$.

2. Let $T=$TAFTPQITR. What is the smaller of the roots of the equation $x^2-57x+T=0$?

3. Let $T=$TAFTPQITR. An ellipse with major axis of length $T+16$ and minor axis of length 8 has foci that are a distance D away from the ellipse's center. Find the value of $\lfloor D \rfloor$, the greatest integer less than or equal to D.

4. Let $T=$TAFTPQITR. The graph of $y=\sin(Tx)$ intersects the x-axis how many times on the interval $[0,2\pi)$?

5. Let $T=$TAFTPQITR. In how many consecutive zeros does $(T^2)!$ end?

6. Let $T=$TAFTPQITR. Let $S(x)=$the sum of the digits of x, and define $R(x)=\begin{cases} S(x), \text{ if } S(x) \text{ is a single-digit number} \\ R(S(x)), \text{ if } S(x) \text{ has more than one digit} \end{cases}$. Find the value of $R\left(\binom{T}{2}\right)$.