Analytic Geometry
Alpha, Round 3
Test #623

1. Write your 6-digit ID# in the I.D. NUMBER grid, left-justified, and bubble. Check that each column has only one number darkened.
2. In the EXAM NO. grid, write the 3-digit Test # on this test cover and bubble.
3. In the Name blank, print your name; in the Subject blank, print the name of the test; in the Date blank, print your school name (no abbreviations).
4. Scoring for this test is 5 times the number correct + the number omitted.
5. You may not sit adjacent to anyone from your school.
6. TURN OFF ALL CELL PHONES OR OTHER PORTABLE ELECTRONIC DEVICES NOW.
7. No calculators may be used on this test.
8. Any inappropriate behavior or any form of cheating will lead to a ban of the student and/or school from future national conventions, disqualification of the student and/or school from this convention, at the discretion of the Mu Alpha Theta Governing Council.
9. If a student believes a test item is defective, select "E) NOTA" and file a Dispute Form explaining why.
10. If a problem has multiple correct answers, any of those answers will be counted as correct. Do not select "E) NOTA" in that instance.
11. Unless a question asks for an approximation or a rounded answer, give the exact answer.

Note: For all questions, answer "(E) NOTA" means none of the above answers is correct.

1. Find the equation of the horizontal line that passes through the point $(-3, 2)$.

   (A) $x = 2$ (B) $y = 2$ (C) $x = -3$ (D) $y = -3$ (E) NOTA

2. Find the distance from the point $(2, -1, 5)$ to the plane $2x + 2y = z + 3$.

   (A) 1 (B) $\sqrt{30}/5$ (C) 2 (D) $\sqrt{30}/10$ (E) NOTA

3. Find the equation of a line passing through $(4, 2)$ and is also perpendicular to the line passing through $(11, 4)$ and $(9, 7)$.

   (A) $y = \frac{2}{3}(x - 9) + 7$ (B) $y = -\frac{3}{2}(x - 9) + 7$

   (C) $y = -\frac{3}{2}(x - 4) + 2$ (D) $y = \frac{2}{3}(x - 4) + 2$ (E) NOTA

4. The graphs of $y = x^2 + 1$ and $y = 2x + 16$ intersect at (a, b) and (c, d). Find the value of $a + b + c + d$.

   (A) 26 (B) 38 (C) 8 (D) 17 (E) NOTA

5. Identify the type (or types) of symmetry for the graph of $3x^4 + xy = 2$ in the plane.

   (A) x-axis (B) y-axis (C) origin (D) both B and C (E) NOTA

6. If quadrilateral ABCD is inscribed in a circle, how many of the statements I-IV below are always true?

   I. $\sin A = \sin C$ II. $\sin A + \sin C = 0$ III. $\cos B + \cos D = 0$ IV. $\cos B = \cos D$

   (A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

7. Find the area of the circle with equation $x^2 + y^2 + 8y = 2x + 20$.

   (A) 20π (B) 48π (C) 30π (D) 37π (E) NOTA

8. Which of the following is true about the graph of $y = f(x) = \frac{x^3-2x^2-29x-42}{x^2-9}$?

   (A) The graph of $y = f(x)$ has a removable discontinuity at $x = -3$.

   (B) The graph of $y = f(x)$ has a jump discontinuity at $x = 3$.

   (C) If the function was re-defined so that $3f(3) = -5$, then $f(x)$ is continuous at $x = 3$.

   (D) The graph of $y = f(x)$ has non-removable discontinuities if $x \in \{-3, 3\}$.

   (E) NOTA

9. In parallelogram $SAND$, $S = (-1, 3)$, $A = (5, 3)$, $N = (c, -8)$, and $D = (4, -8)$. Find the product of all possible values of c.

   (A) -20 (B) -9 (C) 8 (D) 64 (E) NOTA

10. The function $y = f(x)$ has zeroes at $x = -2$ and $x = 6$, and nowhere else. Find the zeroes of $-3f(2 - 2x)$.

    (A) $\{-2, 2\}$ (B) $\{1, 5\}$ (C) $\{-1, 4\}$ (D) $\{-1, -5\}$ (E) NOTA

11. Maria *loves* flowers, especially ones generated from polar equations. If Tony presents Maria with a bouquet of six roses generated by $r = 2\sin(12\theta)$, three roses generated by $r = 2\sin(15\theta)$, and three roses generated by $r = 2\sin(14\theta)$, how many total petals does she have available to pull off in order to play the "He loves me, he loves me not?" game?

    (A) 282 (B) 312 (C) 198 (D) 396 (E) NOTA

12. Find the area of the region bounded by the x-axis and the graph of $y = \sqrt{x^2 - 8x + 16}$ on the interval $-1 \leq x \leq 8$.

    (A) 20 (B) 20.50 (C) 27.50 (D) 40 (E) NOTA

13. Let R be the region bounded by the x-axis, the y-axis, and the line $x + 3y = 9$. What is the positive difference between the volumes generated when R is revolved about the y-axis and when R is revolved about the x-axis?

    (A) 54π (B) 0 (C) 81π (D) 162π (E) NOTA

14. Find the distance between the foci of the conic section whose parametric equations are $x = 2\cos t$ and $y = 3\sin t$.

(A) $2\sqrt{5}$ (B) 1 (C) $2\sqrt{13}$ (D) 2 (E) NOTA

15. Find the point of inflection—that is, the point of maximum rate of growth—in the graph of $y = 8/(2 + 3e^{-x})$.

(A) $(\ln 1.5\,, 4)$ (B) $(\ln 1.5\,, 2)$ (C) $\left(\ln\left(\frac{2}{3}\right), 2\right)$ (D) $(\ln 4\,, 2)$ (E) NOTA

16. How many of statements I-IV below are true for $g(x) = \begin{cases} |x+1| & x \le -2 \\ x+1 & -2 < x < 1 \\ \sqrt{x+3} & 1 \le x \le 6 \\ 6(8-x)^{-1} & 6 < x < 7 \\ 6 & 7 < x \le 10 \end{cases}$?

I. g is continuous from the right at $x = -2$.

II. g is continuous at $x = 7$.

III. g is continuous from the left at $x = 1$.

IV. g is continuous at $x = 9$.

(A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

17. If $f(x) = (|2 - x| + 4)^{-1}$ and $y = g(x) = f(-2x)$, find the range of $g(x)$.

(A) $y \in \left(0, \frac{1}{8}\right)$ (B) $y \in \left(0, \frac{1}{6}\right)$ (C) $y \in \left(0, \frac{1}{5}\right)$ (D) $y \in \left(0, \frac{1}{4}\right)$ (E) NOTA

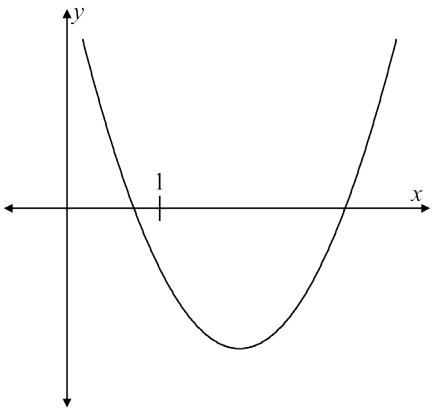
18. The graph of $y = \sin\left(3x + \frac{\pi}{12}\right) - \frac{1}{2}$ crosses the x-axis four times on the interval $0 \le x \le \pi$. If the x-coordinates of these four x-intercepts are $a\pi$, $b\pi$, $c\pi$, and $d\pi$, where $a < b < c < d$, find the value of $a + c$.

(A) 13/18 (B) 7/18 (C) 5/6 (D) 4/9 (E) NOTA

19. Find the amplitude of the graph of $y = 3\sqrt{5}\sin(7x) - 6\sqrt{7}\cos(7x)$.

(A) $6\sqrt{7}$ (B) $\left|3\sqrt{5} - 6\sqrt{7}\right|$ (C) $3\sqrt{5} + 6\sqrt{7}$ (D) $3\sqrt{33}$ (E) NOTA

20. Shown below is a portion of the graph of $y = ax^2 + bx + c$, for constants a, b, and c. How many of the expressions ab, ac, $a + b + c$, and $a - b + c$ have positive value?



(A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA

21. If $y = a$ and $y = b$ are the two horizontal asymptotes of the graph of $y = \frac{8+3^x}{4-3^x}$, find the value of $|a + b|$.

(A) 11 (B) 2 (C) 4 (D) 1 (E) NOTA

22. Find the measure of the positive angle from line L_1 to line L_2, if L_1 is vertical and L_2 has a slope of $1/2$. Measure the angle by going counterclockwise.

(A) $120°$ (B) $150°$

(C) $\text{Arctan}(-2) + 180°$ (D) $\text{Arctan}\left(-\frac{1}{2}\right) + 180°$ (E) NOTA

23. Find the slope of the line that bisects the angle formed by lines L_1 and L_2 if L_1 has a slope of 2 and L_2 is a vertical line.

(A) $2 \pm \sqrt{5}$ (B) 4 (C) $\frac{1\pm\sqrt{5}}{2}$ (D) $1 \pm \sqrt{5}$ (E) NOTA

24. How many of the equations I-IV below are correct symmetric forms of the line passing through the points (2, 2, 4) and (8, 6.5, 2.5)?

I. $\frac{x-2}{4} = \frac{y-2}{3} = \frac{z-4}{-1}$ II. $\frac{x-2}{6} = \frac{y-2}{4.5} = \frac{z-4}{-1.5}$

III. $\frac{x-8}{4} = \frac{y-6.5}{3} = \frac{z-2.5}{-1}$ IV. $\frac{x-8}{12} = \frac{y-6.5}{9} = \frac{z-2.5}{-3}$

(A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

25. Find the equation of the plane through (3, 0, 1) and perpendicular to the line $x = 2t$, $y = 1 - t, z = 4 - 3t$.

(A) $y + 4z = 4$ (B) $3x + z - 3 = 0$

(C) $x + y + 4z = 7$ (D) $2x - y = 3z + 3$ (E) NOTA

26. Find the area of the triangle with vertices at (4,2,4), (10,2, −2), and (2,0, −4).

(A) 27 (B) $24\sqrt{3}$ (C) 36 (D) $18\sqrt{3}$ (E) NOTA

27. Find the volume of the parallelepiped whose edges are represented by the vectors $\vec{a} = 2\mathbf{i} + 3\mathbf{j}$, $\vec{b} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$, and $\vec{c} = 4\mathbf{i} - \mathbf{k}$.

(A) 19 (B) 25 (C) 16 (D) 22 (E) NOTA

28. The ellipse with equation $x^2/9 + y^2/4 = 1$ is rotated counterclockwise about the origin by 45°. The resulting equation can be written in the form

$$a(x')^2 + b(x'y') + c(y')^2 = 72.$$

Find the value of a.

(A) 13 (B) −2 (C) 14 (D) 18 (E) NOTA

29. For each real number m, the parabola $y = (m^2 + 4)x^2 + (m - 2)x - 4m + 2$ passes through the same point (a, b). Find the value of $a^2 + b^2$.

(A) 10 (B) 25 (C) 15 (D) 20 (E) NOTA

30. There are two circles tangent to both the x and y-axes and passes through $(-8,-1)$. Find the sum of the lengths of the two radii.

(A) $16\sqrt{3}$ (B) 16 (C) 18 (D) 18.50 (E) NOTA