The use of a calculator is **not** permitted.
For all questions, choice **E** is "NOTA", meaning "None of These Answers."
For this test, i is defined as $\sqrt{-1}$, and $\text{cis}(\theta) = \cos\theta + i\sin\theta$.

1. There are four boys and two girls on lunch detail. Three of the children will be selected at random to move on to detention. What is the probability that neither girl is selected?

A) $\frac{1}{5}$ B) $\frac{1}{4}$ C) $\frac{1}{3}$ D) $\frac{1}{2}$ E) NOTA

2. The terminal side of an angle β in standard position passes through the point $\left(\frac{3}{4}, 1\right)$. Evaluate $\sec\beta$.

A) $1\frac{1}{4}$ B) $1\frac{1}{3}$ C) $1\frac{2}{3}$ D) $1\frac{3}{4}$ E) NOTA

3. $\begin{bmatrix} 2 & 3 & -4 \\ 2 & 1 & 3 \\ 0 & 2 & -2 \end{bmatrix} \bullet \begin{bmatrix} -1 & 1 & 5 \\ 3 & -1 & 0 \\ 3 & 3 & 2 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & f & g \\ h & j & k \end{bmatrix}$. Find $|c - j|$.

A) 4 B) 6 C) 8 D) 10 E) NOTA

4. Which of the following statements are true?

I. π is a real number.
II. $\langle 1,1 \rangle$ is a unit vector.
III. The function $f(x) = x\sin(x)$ is even.

A) I and II only B) I and III only C) II and III only D) I, II, and III E) NOTA

5. $f(x) = (x-b)^3(x+a)$. Find the product of all distinct x and y intercepts of the graph of $y = f(x)$.

A) a^2b^4 B) $-a^2b^4$ C) a^2b^6 D) $-a^2b^6$ E) NOTA

6. Consider the arithmetic shown to the right, where each of the digits in the set $\{1,2,4,6,8\}$ is represented by a different letter. Evaluate $|R+S-T|$.

$$\begin{array}{cccccc} & P & S & T & S \\ + & N & R & N & P \\ \hline T & S & N & P & R \end{array}$$

A) 5 B) 7 C) 9 D) 11 E) NOTA

7. The graph of the polar equation $r = \dfrac{4}{2-\cos\theta}$ can be best described as _____ .

A) a circle B) an ellipse C) a hyperbola D) a parabola E) NOTA

8. The productivity factor of a company's workforce is given by the function $P(h,w) = \left| \dfrac{1}{\frac{h}{10} - \log(100w)} \right|$, where *w* represents the number of workers, and *h* represents the number of hours that each worker works per week. Of the choices (A,B,C,D) given below, which is the most productive weekly scenario (greatest value of *P*)? Use the approximation $\log 2 \approx 0.3$.

A) 4 workers each working 40 hours B) 5 workers each working 32 hours
C) 8 workers each working 20 hours D) 10 workers each working 12 hours E) NOTA

9. In triangle *MAF*, $m\angle M = 50°$, $AM = 9$, and $FM = 5$. The length *FA* is closest to which integer? Use the approximations shown, if necessary.

$\cos 50° \approx \dfrac{16}{25}$ $\sin 50° \approx \dfrac{7}{9}$

A) 6 B) 7 C) 8 D) 9 E) NOTA

10. The difference between the square of the length of one diagonal of a regular hexagon and the square of the length of another diagonal of the same hexagon is 64. If the area of the hexagon is $x\sqrt{3}$, then find the tens digit of *x*.

A) 2 B) 5 C) 8 D) 9 E) NOTA

11. Find the distance between the lines with equations $y = 2x$ and $y = 2x + 5$.

A) $\sqrt{5}$ B) $\dfrac{5\sqrt{2}}{2}$ C) $\sqrt{21}$ D) 5 E) NOTA

12. Robert wants to buy an item that has a sticker price of $\$Q$. The store is having a "10% off all sticker prices" sale. He must pay the local sales tax of 5% on the item. He will also use a 20% off coupon on the item. What final price, in dollars, will Robert pay for his item, assuming no rounding is necessary?

A) $\dfrac{3}{4}Q$ B) $\dfrac{19}{25}Q$ C) $\dfrac{77}{100}Q$ D) $\dfrac{189}{250}Q$ E) NOTA

13. Consider a triangle with sides measuring 4, 6, and 8. Let *A* be the length of its shortest altitude. When A^2 is expressed as a reduced improper fraction, find the units digit of the sum of its numerator and

denominator.

A) 1 B) 3 C) 7 D) 9 E) NOTA

14. The roots of the function $y = x^3 - 5x^2 - 2x + 24$ are $p, q,$ and r, where $p < q < r$. Evaluate $|2p - q + r|$.

A) 2 B) 3 C) 4 D) 5 E) NOTA

15. $f(x) = \sqrt{\text{Cos}^{-1}(x)} + \sqrt{\dfrac{1-\sin(x)}{2}}$. What percentage of the domain of f is positive?

A) 25% B) 50% C) 75% D) 100% E) NOTA

16. Without her graphing calculator, Emily wants to sketch the graph of the polar equation $r = \left(\dfrac{5\pi}{4} - \theta\right)\sin\theta$. She only wants to graph a few guiding points, so she decides to divide the interval $[0, 2\pi]$ into seven subintervals of equal length and use the set of all the right endpoints of each subinterval as the domain that she will graph. How many of the points that she will graph lie in Quadrant I ?

A) 0 B) 1 C) 2 D) 3 E) NOTA

17. Let $M = \dfrac{5\pi}{3}$, $k = e^{Mi}$, and $\sqrt{\sqrt{k}} = \text{cis}(x°)$, where $0 \le x < 360$. Find x.

A) 15 B) 30 C) 75 D) 120 E) NOTA

18. Consider the three-digit positive number, x, with the following properties:

- When the digits of x are reversed, a number y is obtained such that $200 < y - x < 300$.
- When just the first two digits of x are swapped, a number z is obtained such that $x - z = 450$.
- $y + z = 1085$

Find the sum of the digits of the sum of the digits of x.

A) 6 B) 7 C) 8 D) 9 E) NOTA

19. Consider the polynomial function $f(x) = Ax^5 + Bx^4 - 3x^3 - x^2 - 9$, where $A > 0 > B$. Let p be the number of positive real roots, let n be the number of negative real roots, and let z be the number of nonreal roots.

Which of the following inequalities could be true?

A) $n > p > z$ B) $n > z > p$ C) $z > n > p$ D) $z > p > n$ E) NOTA

20. The graphs of the functions $f(x) = \left|1 - \left|x - 2\right|\right|$ and $g(x) = \frac{x+1}{2}$ intersect how many times?

A) 1 B) 2 C) 3 D) 4 E) NOTA

21. Shifting the graph of $y = \cos\left(\frac{\pi}{4} + x\right)$ in what manner will yield the graph of $y = \sin x$?

A) $\frac{3\pi}{4}$ units left B) $\frac{\pi}{4}$ units left C) $\frac{\pi}{4}$ units right D) $\frac{3\pi}{4}$ units right E) NOTA

22. Consider the two well-known sequences of positive integers S_1 and S_2. Neither sequence is arithmetic or geometric. The sequence 3,4,7,10,16,21,30,40,... is the termwise sum of S_1 and S_2. What is the 10^{th} term of this sequence?

A) 62 B) 69 C) 75 D) 84 E) NOTA

23. The trinomial $12x^2 - 11x - 15$ can be factored as $(Ax + B)(Cx + D)$, where $A > C > 0$ and $\{A, B, C, D\}$ is a subset of the set of all integers. What is *B*?

A) -5 B) -3 C) 3 D) 5 E) NOTA

24. $4\sin\left(\frac{13\pi}{12}\right) = \sqrt{A} - \sqrt{B}$, where *A* and *B* are positive integers. Evaluate $2A + B$.

A) 10 B) 11 C) 13 D) 14 E) NOTA

25. $\left(\lim_{x \to 0} \frac{\sin x}{x}\right) + (0!) =$

A) 0 B) 1 C) 2 D) π E) NOTA

26. Some values of $f(x)$, which is not continuous, are given in the table shown below.

Let $A = f^{-1}(2)$, if f is a one-to-one function.
Let $B = f(2)$, if f is an odd function.
Evaluate $|2A - B|$.

x	–2	–1	3
$f(x)$	1	2	–2

A) 1 B) 3 C) 5 D) 7 E) NOTA

27. Walter practices somewhat misguided oral hygiene on three occasions per day (morning, afternoon, and night), using the same schedule every week. On each occasion, he does exactly one of the following: brushes his teeth, flosses his teeth, or uses mouthwash. He performs each action once per day, but he never performs the same action at the same time three days in a row.

He never uses mouthwash at night, and he is more likely to brush his teeth on any given night than to floss. If Walter uses mouthwash in the morning on every day that begins with the letter "T", and he brushes his teeth in the afternoon on every day that begins with the letter "S", then how many of the following statements must be true?

- Walter's routine is identical on Monday and Wednesday.
- Walter brushes his teeth on Thursday morning.
- Walter brushes his teeth on Friday morning.
- Walter uses mouthwash in the morning four times per week.

A) 1 B) 2 C) 3 D) 4 E) NOTA

28. Let the point **F** be one focus of an ellipse. The distance from **F** to an endpoint of the ellipse's minor axis is 7. **F** is *x* units away from one endpoint of the ellipse's major axis; **F** is $(x+6)$ units away from the other endpoint of the ellipse's major axis. If the minor axis of the ellipse is $\sqrt{A}$ units long, then find the units digit of *A*.

A) 0 B) 2 C) 3 D) 6 E) NOTA

29. Find the coefficient of the x^5y^3 term in the expansion of $\left(\frac{x}{4}+2y\right)^8$.

A) $\frac{7}{16}$ B) $\frac{35}{8}$ C) 28 D) 112 E) NOTA

30. Which of the following is **not** a valid identity when both sides of the equation are defined?

A) $\frac{\sin\theta\cos\theta}{\cos^2\theta-\sin^2\theta}=\frac{\tan\theta}{1-\tan^2\theta}$ B) $\cos\beta(\tan\beta+\cot\beta)=\csc\beta$

C) $\sec^4 x-\sec^2 x=\tan^4 x+\tan^2 x$ D) $1-2\sin^2\lambda=2\cos^2\lambda-1$ E) NOTA