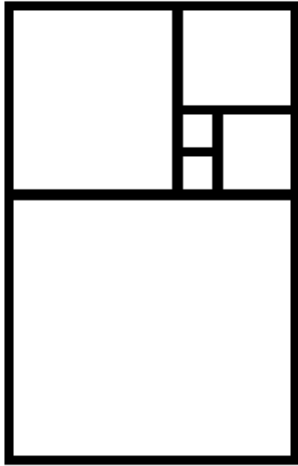Note: For all questions, answer "(E) NOTA" means none of the above answers is correct.

1. A sphere has radius 12. Ignoring units, what is the ratio of its volume to its surface area?
   (A) 2   (B) 3   (C) 4   (D) 5   (E) NOTA

2. An equiangular hexagon ABCDEF has AB = 3, BC = 4, CD = 2, DE = 5, EF = 2, FA = 4. Find the area of the hexagon.

   (A) $16\sqrt{2}$ (B) $16\sqrt{3}$ (C) 18   (D) $18\sqrt{3}$ (E) NOTA

3. Three tennis balls of radius 3 fit perfectly into a cylindrical can, the tennis balls stacked vertically. What is the volume of space in the can not taken up by the balls?
   (A) 48π (B) 54π (C) 60π (D) 64π (E) NOTA

4. Circle O has radius 6. Lines through a point C outside of circle O meet and are tangent to circle O at points A and B. If the measure of angle ACB is 60 degrees, what is the area of quadrilateral ACBO?

   (A) $12\sqrt{3}$ (B) $24\sqrt{2}$ (C) 30   (D) $36\sqrt{3}$ (E) NOTA

5. Find the area of a convex pentagon with vertices (2,2), (-3,0), (-1, -2), (2,-1), and (-2, 2)?

   (A) 15   (B) 15.5   (C) 16   (D) 16.5   (E) NOTA

6. The region bounded by the coordinate axes and the portion of the line y = -2x + 4 in the first quadrant is rotated around the y-axis. What is the volume of the resulting solid?

   (A) 16π (B) 12π (C) 24π (D) 18π (E) NOTA

7. A circle circumscribes a triangle with side lengths 14, 48, and 50. What is the area of the circle?
   (A) 576π (B) 600π (C) 625π (D) 676π (E) NOTA

8. The area of a triangle bounded by the graphs of y = |x| - b and y = x/2 is 12, where b > 0. What is the value of b?

   (A) 2   (B) 3   (C) 4   (D) 5   (E) NOTA

9. A circle of radius 8 has a chord of length $8\sqrt{3}$, dividing the circle into two regions. What is the area of the smaller region that the chord creates in the circle?

   (A) $24\pi - 12\sqrt{3}$ (B)$64\pi/3 – 16\sqrt{3}$(C) $30\pi – 15\sqrt{2}$ (D) $32\pi/3 – 8\sqrt{3}$ (E) NOTA

10. A circle is inscribed inside an equilateral triangle of length 1. The circle circumscribes another equilateral triangle, another circle is inscribed inside that equilateral triangle, and so on. What is the sum of the areas of all the circles drawn?

    (A) π/6   (B) π/10   (C) π/12   (D) π/9   (E) NOTA

11. Triangle ABC has AB = 13, BC = 14, CA = 15. Point D is drawn on CA such that BD bisects angle ABC. What is the ratio of the area of triangle CBD to the area of triangle ABD?

    (A) 14/13   (B) 13/15   (C) 15/14   (D) 1   (E) NOTA

12. Two distinct points X and Y are fixed in 3-space. Suppose we want to choose a point Z such that the triangle XYZ has area equal to 2013. What is the locus of points Z satisfying this property?

    (A) A line.   (B) Two lines.   (C) A circle.   (D) A sphere.   (E) NOTA

13. Parallelogram ABCD has AB = CD = 4, BC = DA = 5, and $m\angle BAD = 60°$. The midpoints of AB, BC, CD, and DA are P, Q, R, and S, respectively. What is the area of quadrilateral PQRS?

    (A) 6   (B) $5\sqrt{3}$ (C) $6\sqrt{2}$ (D) 10   (E) NOTA

14. A cup in the shape of a right circular cone has radius 6 and height 8, with the vertex on the bottom, and the top of the cone parallel to the horizontal plane. The cup is initially filled with water with a volume of 12π. If we continue filling the cup with water until the water level increases by one unit, what is the volume of water now in the cup?

    (A) 225π/12   (B) 375π/16   (C) 24π   (D) 28π   (E) NOTA

15. Beginning with two unit squares, a rectangle is constructed by building a sequence of squares spiraling around the currently formed rectangle, as in the figure below. If at the end of the construction there are 11 squares, what is the total area of the rectangle?

(A) 4895   (B) 9196   (C) 11102   (D) 12816   (E) NOTA

16. A triangle with integer side lengths has perimeter 14. What is the smallest area this triangle could possibly have?

(A) $2\sqrt{14}$ (B) $2\sqrt{21}$ (C) $3\sqrt{6}$ (D) $2\sqrt{17}$ (E) NOTA

17. Triangle ABC has AB = 6, BC = 6, AC = $4\sqrt{3}$. Segments BA and BC are extended through A and C, respectively. A circle O is drawn tangent to the extension of BA, the extension of BC, and AC at D, E, and F respectively. What is the area of quadrilateral BDOE?

(A) $21\sqrt{6}$ (B) $15\sqrt{3} + 4\sqrt{6}$ (C) $24\sqrt{2} + 12\sqrt{6}$ (D) $15\sqrt{6} + 6\sqrt{3}$ (E) NOTA

18. Which of the following matrices preserves area?

(A) $\begin{bmatrix} 7 & 4 \\ 5 & 3 \end{bmatrix}$ (B) $\begin{bmatrix} 6 & 4 \\ 4 & 3 \end{bmatrix}$ (C) $\begin{bmatrix} 5 & 7 \\ 4 & 6 \end{bmatrix}$ (D) $\begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$ (E) NOTA

19. The two foci of an ellipse are located at (8,3) and (2, 11). If (64/5, -17/5) is located on the perimeter of the ellipse, what is the area of the ellipse?

(A) 132π (B) 156π (C) 160π (D) 172π (E) NOTA

20. What is the surface area of a regular icosahedron with side length 8?

(A) $256\sqrt{3}$ (B) $320\sqrt{3}$ (C) $384\sqrt{3}$ (D) $400\sqrt{2}$ (E) NOTA

21. Consider a circle with center O and radius 6. There is a point C outside the circle. Lines through C are tangent to the circle at points A and B. If the distance between O and C is 10, what is the area of the quadrilateral OACB?

    (A) 40   (B) 45   (C) 48   (D) 50   (E) NOTA

22. A cube has side length 8. Four distinct points are chosen from the centers of the cubes, not all coplanar. What is the volume of the tetrahedron whose vertices are these points?

    (A) 20   (B) 21   (C) 22   (D) 23   (E) NOTA

23. Isosceles triangle ABC has $m\angle B$ = 90°. Points D and E are on BC such that AD and AE trisect angle A, and D is between B and E. What is the ratio of the area of ABD to AEC?

    (A) 1   (B) $\sqrt{3}$ -1   (C) $\frac{2+\sqrt{3}}{3}$ (D) $\frac{3-\sqrt{3}}{2}$ (E) NOTA

24. Three unit circles are mutually externally tangent to each other. What is the area of the smaller circle that is tangent to all three circles?

    (A) $\pi(3 - 2\sqrt{2})$ (B) $\pi\frac{9-6\sqrt{2}}{2}$ (C) $\pi\frac{7-4\sqrt{3}}{3}$ (D) $\pi/6$ (E) NOTA

25. Isosceles right triangle ABC has $m\angle A$ = 90°, and AB = AC = 6. A point D on the circumcircle of ABC is chosen at random. A convex quadrilateral is formed by the points A, B, C, and D (not necessarily in that order). What is the probability that the quadrilateral formed has area less than 27?

    (A) 2/3   (B) 3/4   (C) 4/5   (D) 5/6   (E) NOTA

26. Given a regular octagon, how many triangles of distinct area can be created from choosing 3 distinct vertices of the octagon?

    (A) 3   (B) 4   (C) 5   (D) 6   (E) NOTA

27. A rectangular prism has a space diagonal of length 60 and the sum of the lengths of all its sides is 336. What is the prism's surface area?

    (A) 2880   (B) 3150   (C) 3576   (D) 3675   (E) NOTA

28. A notecard with dimensions of 3 inches by 5 inches is folded along its diagonal. What is the area of the resulting figure?

    (A) 99/10 in²   (B) 48/5 in²   (C) 10 in²   (D) 147/10 in²   (E) NOTA

29. What is the area of the convex polygon formed in the Argand plane with vertices that are the roots of the equation $x^8 = -1$?

   (A) 1   (B) 2   (C) $2\sqrt{2}$ (D) $3\sqrt{2}$ (E) NOTA

30. A 12-hour analog clock shows 2:45. Of the smaller of the two areas, what is the fraction of the area of the clock the arms subtend?

   (A) 7/15   (B) 9/20   (C) 23/48   (D) 15/32   (E) NOTA