Note: For all questions, answer "(E) NOTA" means none of the above answers is correct.

1. Assume that a cart on a circular Ferris wheel begins at the bottom, zero feet off the ground. The cart then completes a 100π ft cycle in 6 minutes. How many seconds after starting is the cart 75 feet above the ground for the first time?

   (A)90 (B)120 (C)150 (D)180 (E) NOTA

2. The faces of a regular cube are randomly numbered with a distinct integer from 1 to 6. What is the probability that no pair of adjacent faces contain consecutive integers?

   (A)$\frac{1}{6}$ (B)$\frac{1}{3}$ (C) $\frac{5}{12}$ (D) $\frac{1}{2}$ (E) NOTA

3. What is the sum of the tens and units digit of $(15 \cdot 25 \cdot 35 \cdot 45)^2$?

   (A)6 (B)7 (C)8 (D)12 (E) NOTA

4. For what value of a is the area enclosed by the polar curve $r = a \sin\theta$ equal to 25, where $0 \leq \theta \leq \pi$?

   (A)$\frac{5}{2}$ (B)$\frac{5}{\sqrt{\pi}}$ (C)$\frac{10}{\sqrt{\pi}}$ (D)5 (E) NOTA

5. If x and y are integers such that $y = \frac{6}{x} - 2$, what is the greatest possible value of xy?

   (A)2 (B) 4 (C) 10 (D) 12 (E) NOTA

6. Let x, y, and z be integers such that $x < y < z$. If $xy = 24$ and $yz = 42$, what is the least possible value of $x + y + z$?

   (A)17 (B) 18 (C) 19 (D) 20 (E) NOTA

7. What is the period of the following combination of trigonometric functions?

$$f(t) = \sin(28\pi t) + \sin(36\pi t) + \cos(4\pi t)$$

   (A)$\frac{1}{2}$ (B)2 (C)2π (D)4π (E) NOTA

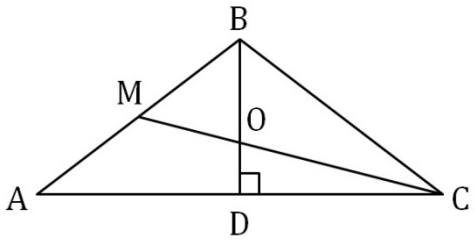
8. Evaluate: $\sin\left(\frac{\pi}{16}\right)\cos\left(\frac{\pi}{16}\right)\cos\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{4}\right)$

(A)$\frac{\sqrt{2}}{2}$ (B)$\frac{1}{8}$ (C)$\frac{1}{4}$ (D)1 (E) NOTA

9. The set S consists of all 4-digit positive integers whose consecutive digits form primes (for example, in the number 1379, the consecutive digits, 13, 37, and 79, are all prime) and whose leading digit is 2. What is the sum of the numbers in S?

(A) 10580 (B)12951 (C) 15330 (D)17641 (E) NOTA

10. In the following diagram, ABC is an isosceles triangle with $AB = BC$. Let M and D be the midpoints of AB and AC respectively. If O is the intersection of BD and MC, what is the ratio of BO to OD?



(A)$\frac{3}{2}$ (B)2 (C)$2\sqrt{2}$ (D)3 (E) NOTA

11. Let ABC be a triangle with integer side lengths such that $AB = 3$ and $BC = 5$. What is the sum of all values of AC such that $\angle B$ is obtuse?

(A)6 (B)7 (C)8 (D)9 (E) NOTA

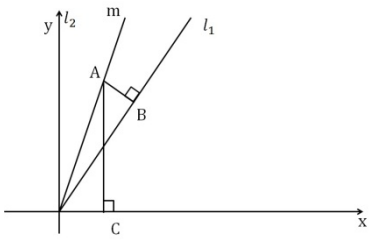
12. Let a and b be distinct positive primes less than 100. What is the probability that their product has a remainder of 1 when divided by 5?

(A)$\frac{7}{50}$ (B)$\frac{26}{150}$ (C)$\frac{59}{300}$ (D)$\frac{23}{100}$ (E) NOTA

13. Integers a, b, and c are randomly chosen such that $0 < a, b, c \leq 6$. What is the probability that the vectors $< 1, a >$ and $< b, c >$ are parallel?

(A)$\frac{5}{108}$ (B) $\frac{1}{18}$ (C) $\frac{7}{108}$ (D) $\frac{15}{216}$ (E) NOTA

14. In the following diagram, line l_1 lies at 60^o to the x-axis, l_2 lies along the y-axis, and line m bisects the angle made by l_1 and l_2. Point A is located on m a distance 5 from the origin, and points B and C are feet of the perpendicular from A to l_1 and the x-axis respectively. What is $AB + AC$?



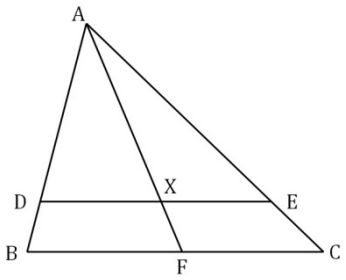
(A)5 (B)$\frac{5\sqrt{6}}{2}$ (C)6 (D)$5\sqrt{2}$ (E) NOTA

15. For $\frac{\pi}{6} \leq \theta \leq \frac{\pi}{2}$, what is the maximum possible real root of

$$0 = x^2 - 2\cos(\theta)\, x + \frac{\sin^2(2\theta)}{4}?$$

(A) $\frac{3}{2}$ (B) $\frac{2\sqrt{3}+3}{4}$ (C) 2 (D)$\frac{3+\sqrt{3}}{2}$ (E) NOTA

16. In the following diagram, $\overline{AF}$ intersects $\overline{DE}$ at X. If $\overline{DE}$ is parallel to $\overline{BC}$ and $\frac{DE}{BC} = \frac{4}{5}$, what is the ratio of the area of triangle ADE to the area of *parallelogram* $DFCX$?



(A)$\frac{16}{25}$ (B)$\frac{16}{9}$ (C)$\frac{18}{5}$ (D)4 (E) NOTA

17. Points A, B, C, and D are chosen as distinct points with integer coordinates in the first quadrant of the Cartesian plane. If no point lies on the x or y-axis and each point lies on the circumference of a circle of radius 25 centered at the origin, what is the area of quadrilateral $ABCD$?

(A) 44 (B) $44\sqrt{2}$ (C) 88 (D) $44\sqrt{5}$ (E) NOTA

18. Let θ be the smaller angle formed between the lines $y = \frac{1}{4}x + 5$ and $y = 4x - 3$. What is $\sin(\theta)$?

(A) $\sin(50°)$ (B) $\frac{8}{17}$ (C) $\frac{1}{2}$ (D) $\frac{15}{17}$ (E) NOTA

19. Define a recursive function for $n \geq 3$ as follows:

$$f(n) = \left\lfloor \frac{n-1}{n} f(n-1) \right\rfloor + \left\lfloor \frac{n-2}{n} f(n-2) \right\rfloor$$

($\lfloor x \rfloor$ = the greatest integer less than or equal to x). If $f(6) = f(7) = 2$, what is $f(14)$?

(A) 26 (B) 28 (C) 42 (D) 68 (E) NOTA

20. For what value of x is $\sin^{-1}\left(\frac{1}{7}\right) + \cos^{-1}(x) = \pi$?

(A) $-\frac{4\sqrt{3}}{7}$ (B) $-\frac{1}{7}$ (C) $\frac{3\sqrt{5}}{7}$ (D) $\frac{4\sqrt{3}}{7}$ (E) NOTA

21. If $\sin(\theta_1 + \theta_2) = \frac{24}{25}$, find the maximum value of $(\sin(\theta_1) + \cos(\theta_1))(\sin(\theta_2) + \cos(\theta_2))$.

(A) $\frac{49}{25}$ (B) 2 (C) 4 (D) $\frac{168}{25}$ (E) NOTA

22. For $-180^o \leq \theta \leq 180^o$, what is the range of θ such that

I. $\sin(\theta) \leq \cos(\theta)$,

II. $\cot(\theta) \leq \tan(\theta)$, and

III. $\sin(\theta)^2 \geq \cos(\theta)$?

(A) $-135^o \leq \theta \leq -90^o$ (B) $-90^o \leq \theta \leq -45^o$ (C) $0^o \leq \theta \leq 90^o$
(D) $135^o \leq \theta \leq 180^o$ (E) NOTA

23. Mark passes a rope through his dog's collar such that the dog can run anywhere along the length of the 10m (meter) rope. The ends of the rope are tied down at points 6m apart. What is the area, in square meters, of the region of land over which the dog can travel?

(A) 30 (B) 60 (C) 20π (D) 30π (E) NOTA

24. What is the least value of α in radians, such that $|\sin(x) + 2\cos(x)| = 2$ has at least 7 solutions for $0 \le x \le \alpha$?

(A)$\frac{\pi}{2}$ (B)$\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ (C) 2π (D) 3π (E) NOTA

25. Let y' be the graph of $y = -x^2 + 2x + 2$ rotated 90^o counterclockwise about the origin. At how many points do the graphs of y and y' intersect?

(A)0 (B)2 (C)3 (D)4 (E) NOTA

26. Determine the least possible positive integer value of x, such that

$$\sum_{i=0}^{x}\left[i + \sin\left(\frac{\pi}{4}i\right) + \cos\left(\frac{\pi}{4}i\right)\right] \ge 350\,.$$

(A) 19 (B)25 (C)26 (D)27 (E) NOTA

27. Let the function $f(x) = \frac{1}{(1-x)^2}$ be rewritten as an infinite polynomial of the form $f(x) = a_0 + a_1x + a_2x^2 + \cdots$ for $|x| < 1$. What is a_4?

(A) 0 (B) 1 (C) 3 (D) 4 (E) NOTA

28. Let u, v, and w be real roots of the polynomial $x^3 - 4x^2 - 7x + 10$. Determine

$$\tan[\arctan(u) + \arctan(v) + \arctan(w)].$$

(A)$\sqrt{3}$ (B)$\frac{5}{2}$ (C) $\frac{7}{4}$ (D)2 (E) NOTA

29. Define the *distance* between any two cards in a deck of cards to be *the number of cards between the two cards* (e.g. the distance between the top and bottom cards of a 52-card deck is 50). The distances between every pair of four Aces in a standard 52-card deck are x, 8, 12, 22, 26, and 35. What is the sum of all possible values of x?

(A) 16 (B) 19 (C) 25 (D) 26 (E) NOTA

30. At how many points do the graphs of $f(x) = \sin\left(\frac{\pi}{2}x\right)$ and $g(x) = \sin\left(\frac{\pi}{2}x^2\right)$ intersect for $0 \le x \le 4$?

(A)4 (B)7 (C)9 (D)12 (E) NOTA