Practice Round Alpha School Bowl
Mu Alpha Theta National Convention 2013

P1. What is the common difference of the arithmetic sequence $-10, 23, \ldots$?

P2. Find the sum of the digits of the base ten representation of 2^{15}.

P3. Find the *smaller* value of k satisfying $k^2 - 16 = 0$.

P4. Evaluate: $\csc 330°$

P5. Let A, B, C, and D be the answers to problems P1, P2, P3, and P4, respectively.
Evaluate: $(A - B)^{D-C}$

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Round #1 Alpha School Bowl
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1. Find the largest integer x such that $|x-2| < |x-6|$.

2. Find the period of the graph of $y = \left|\sin\left(\frac{\pi x}{6}\right)\right|$.

3. Evaluate: $8\sqrt{2}\cos\frac{7\pi}{4} - 6\sqrt{3}\sin\frac{4\pi}{3}$

4. If $4^x - 4^{x-1} = 24$, find the value of $(2x)^{2x}$.

5. Let A, B, C, and D be the answers to problems 1, 2, 3, and 4, respectively.
Evaluate: $AB - C + D$

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Round #2 Alpha School Bowl
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6. How many digits are there in the binary representation of 5566?

7. Let $ABCD$ be a square, with E and F the midpoints of AB and BC, respectively. If $m\angle EDF = \theta$, find $\sin\theta$.

8. Given that $\cos x = \frac{\sqrt{3}}{5}$, find the value of $484\cot^2 x$.

9. If f varies jointly as g^2 and h, and $f = 128$ when $g = 4$ and $h = 2$, find f when $g = 3$ and $h = 6$.

10. Let A, B, C, and D be the answers to problems 6, 7, 8, and 9, respectively.
Evaluate: $AB(D - C)$

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Round #3 Alpha School Bowl
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11. Let $L(x)$ be a linear function of positive slope and $I(x)$ be the inverse of $L(x)$. Given that $L(x) = 4I(x) + 3$ for all real x, find the value of $L(10)$.

12. Given that $\cos(2x) = \frac{3}{7}$, the value of $\cos^2 x = \frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

13. Find, in radians, the value of $\text{Arcsin}(\sin\left(\frac{11\pi}{6}\right))$. Recall that $-\frac{\pi}{2} \leq \text{Arcsin } u \leq \frac{\pi}{2}$ for $u \in [-1, 1]$.

14. If $f(x) = 3x - 4$ and $g(x) = \sqrt{x+2}$, evaluate: $f(g^{-1}(5))$

15. Let A, B, C, and D be the answers to problems 11, 12, 13, and 14, respectively.
Evaluate: $AB \cot^2 C + 10D$

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Round #4 Alpha School Bowl
Mu Alpha Theta National Convention 2013

16. Solve for x: $\frac{x+3}{x+5} = \frac{x+1}{x+2}$

17. Evaluate: $\sin 20° (\tan 10° + \cot 10°)$

18. Find the area enclosed by the polar graph $r = 10 \cos \theta$.

19. What is the product of the positive solutions to $x^3 - 7x + 6 = 0$?

20. Let A, B, C, and D be the answers to problems 16, 17, 18, and 19, respectively.
Evaluate: $D\left(A + \sin \frac{C}{B}\right)$

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Round #5 Alpha School Bowl
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21. Find the sum of the third hexagonal number and the fourth octagonal number.

22. If (x, y) is an ordered pair of real numbers such that $x^2 + y^2 = 1$, find the maximum value of $2(x + y)^3$.

23. If $\sin 53° = k$, find the smallest angle θ greater than $775°$ such that $\sin \theta = k$. Express your answer in degrees.

24. Let k be the smallest number greater than 1 such that $\log_k 3 \leq 4$. Find the value of k^8.

25. Let A, B, C, and D be the answers to problems 21, 22, 23, and 24, respectively.
Evaluate: $(B + C)\left(A - \sum_{j=1}^{D+1} j\right)$

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Round #6 Alpha School Bowl
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26. Let vectors $\mathbf{a} = [-1, 0, 1]$, $\mathbf{b} = [3, 4, 3]$, and $\mathbf{c} = [2, -3, 2]$. If $[-6, -17, 6] = c_1\mathbf{a} + c_2\mathbf{b} + c_3\mathbf{c}$, find the value of $c_1c_2c_3$.

27. Find the sum of all $x \in [0, 2)$ such that $\cos(3\pi x) = \sin(2\pi x)$.

28. What is the amplitude of the graph of $y = -2013\sin\left(2x\sqrt{2\pi} - 5e^{-500} + 17x\right)$?

29. Find the area of the region in the plane bounded by the graphs of $(x-3)^2 + (y-12)^2 = 144$ and $(x+4)^2 + (y+12)^2 = 169$.

30. Let A, B, C, and D be the answers to problems 26, 27, 28, and 29, respectively.
Find the determinant of the matrix $\begin{bmatrix} C & B \\ A & D \end{bmatrix}$.

Round #6 Alpha School Bowl
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Round #7 Alpha School Bowl
Mu Alpha Theta National Convention 2013

31. In the Cartesian plane, let O be the origin, $Q = (5, 0)$ and P a point on the circle with equation $x^2 + y^2 = 36$. As P goes along the entire circumference of the circle, the centroid of triangle OPQ traces out a curve C. Find the area enclosed by C.

32. The cosine of the smallest angle in a triangle with side lengths 6, 7, 8 is m/n where m and n are relatively prime positive integers. Find $m + n$.

33. If $f(x) = \sin x$, $g(x) = \cos x$, and $h(x) = f(x)g(x)$, evaluate: $h(\pi/8)$

34. Function f has $f(4096^2) = -30$, $f(8^4) = 100$, $f(81^3) = -51$, and $f(25^6) = 12$. If the domain of f is $D = \{4096^2, 8^4, 81^3, 25^6\}$ and m is the smallest element in D, find $f(m)$.

35. Let A, B, C, and D be the answers to problems 31, 32, 33, and 34, respectively.
Evaluate: $\frac{AD}{\pi}(\sin 15° + C)^2 + B$

Round #7 Alpha School Bowl
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Round #8 Alpha School Bowl
Mu Alpha Theta National Convention 2013

36. Given that $M = \begin{bmatrix} -29 & -20 \\ 42 & 29 \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$, find the sum of the entries of M^{10}.

37. Find the number of times the line $y = \frac{x}{100\pi} - 1$ intersects the graph of $y = \sin x$.

38. Let α and β be the measures of two consecutive angles of a parallelogram. Evaluate: $\cos(\alpha + \beta)$

39. Find the sum of the digits of $(10 - 1)(10^2 - 1)(10^4 - 1)(10^8 - 1)$ when written in base 10.

40. Let A, B, C, and D be the answers to problems 36, 37, 38, and 39, respectively.
Evaluate: $\frac{B-C}{A} + D$

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Round #9 Alpha School Bowl
Mu Alpha Theta National Convention 2013

41. In convex quadrilateral $ABCD$, $|AB| = \sqrt{6}$, $|BC| = 5 - \sqrt{3}$, $|CD| = 6$, $m\angle ABC = 135°$, and $m\angle BCD = 120°$. Find the length of AD in simplest radical form.

42. Let $P(x)$ be the second-degree minimal polynomial with integer coefficients (and positive leading coefficient) such that $P(\cos(\pi/5)) = 0$. Evaluate: $P(-1)$

43. The exterior angles of a dodecagon have measures $\theta_1, \theta_2, \ldots, \theta_{12}$.
Evaluate: $\sin(\theta_1 + \theta_2 + \ldots + \theta_{12})$

44. The determinant of $M = \begin{bmatrix} -1 & -2 & -5 \\ 0 & -1 & 4 \\ 4 & x & 5 \end{bmatrix}$ is $4x - 47$. If the entry in the second row, second column of M^{-1} is equal to 15, find the value of x.

45. Let A, B, C, and D be the answers to problems 41, 42, 43, and 44, respectively.
Evaluate: $A^2 + B^2 + C^2 - D^2$

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45. Let A, B, C, and D be the answers to problems 41, 42, 43, and 44, respectively.
Evaluate: $A^2 + B^2 + C^2 - D^2$

Round #10 Alpha School Bowl
Mu Alpha Theta National Convention 2013

46. If $f(x) = \log_2\left(1 + \sqrt{8x+1}\right) - 2$, evaluate: $f(6) + f(28) + f(496) + f(8128) + f(33550336)$

47. The side lengths of an isosceles triangle with no right angles are csc x, sec x, and cot x. Find the largest possible value of csc x.

48. Find the period of the graph of $y = \sin^2(14x)$.

49. How many 4-digit positive integers in base 10 have 5 as a digit immediately followed by a 3 (as the next digit)?

50. Let A, B, C, and D be the answers to problems 46, 47, 48, and 49, respectively.
Evaluate: $\frac{A(B^2-B)\pi}{C} + D$

Round #10 Alpha School Bowl
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