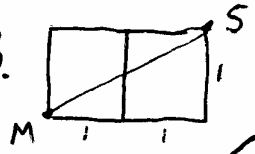
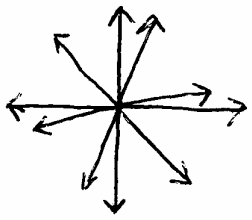
1. $5^2 = 25$ possible combinations
$4^2 = 16$ without a 5
$\frac{25-16}{25} = \boxed{\frac{9}{25}}$

2. 2W OR 2
2R OW 2
1R 1W 2
$\boxed{6}$

3. 

$d = \sqrt{2^2+1^2} = \boxed{\sqrt{5}}$

4. They can't both occur, as both have an L.
For each, any arrangement of the word & the 21 other letters is fine.
$P = P(A) + P(B) = 2(P(A)) = \boxed{\frac{2 \cdot 22!}{26!}}$

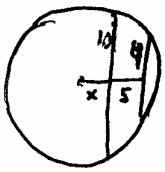
5. 
Three pairs cancel $\Rightarrow \boxed{0}$

6. $\frac{\ln 81}{\ln 3} \quad \frac{\ln 125}{\ln 9} \quad \frac{\ln 16}{\ln 25}$
$\frac{4\ln 3}{3\ln 3} \quad \frac{3\ln 5}{2\ln 5} \quad \frac{4\ln 2}{2\ln 5}$
$\frac{4 \cdot 3 \cdot 4}{3 \cdot 2 \cdot 2} = \boxed{4}$

7. $(\log_a b)(\log_c b)(\log_b a)(\log_a c)$
$\log_a c^{\log_c b} = \boxed{\log_a b}$

8. Hope the solution looks like $a+b\sqrt{3}$.
Squaring gives $a^2+3b^2 = 259$
$2ab\sqrt{3} = 72\sqrt{3}$
$ab = 36$
$a = 4\ b = 9 \Rightarrow \boxed{4+9\sqrt{3}}$

9. roots are r, s, & t
$r+s+t = \frac{-0}{4}$
$rs+st+rt = \frac{-1}{2} = \frac{-2}{4}$
$rst = \frac{-1}{4}$
$r^2+s^2+t^2 = (r+s+t)^2 - 2(rs+st+rt) = 0^2 - 2\left(\frac{-1}{2}\right) = \boxed{1}$

10. 

$r^2 = x^2+100 = (x+5)^2+$ ~~16~~

$x^2+100 = x^2+10x+25+$ ~~16~~

$59 = 10x$

$\frac{59}{10} = x$

11. $\vec{A} = [-3,-2,8]$

$\vec{B} = [-5,-5,1]$

$\vec{A}\times\vec{B} = [38,-37,5]$

$38x-37y+5z=-88$

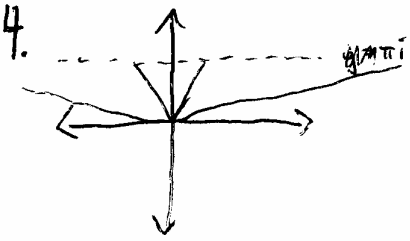
12. $p=20 \Rightarrow$ longest side ≤ 10

9,9,2 8,8,4 7,7,6
9,8,3 8,7,5
9,7,4 8,6,6
9,6,5

8

13. 2 cases: $\binom{3}{3}(1)^3+\binom{3}{1}(2x^2)^1\left(-\frac{3}{x}\right)^2$

$1+54 = 55$

14. 

$\angle(n+\pi i)^4 = 90°, 270°, 450°, 630°$

$\angle(n+\pi i) = 22.5, 67.5, 112.5, 157.5$

4

15. $7560 = 2^3\cdot 3^3\cdot 5^1\cdot 7^1$

We require one 2 & one 7,
leaving $2^2\cdot 3^3\cdot 5^1$

$(2+1)(3+1)(1+1) = 24$

16. $A+B = -8+2i$

$AB = \frac{2B}{A^2} \Rightarrow (A^3-2)B = 0$

if $B=0$, $A = -8+2i$

if not, A is a complex cube root of 2
- sum to 0.

17. $x^{12}-y^{12} = (x^6+y^6)(x^6-y^6)$

$= (x^2+y^2)(x^4-x^2y^2+y^4)(x^2-y^2)(x^4+x^2y^2+y^4)$

$= (x^2+y^2)(x^4-x^2y^2+y^4)(x+y)(x-y)$
$(x^2+y^2+xy)(x^2+y^2-xy)$

18. $3+7+2\sqrt{21}$

$10+2\sqrt{21}$ $4.5 < \sqrt{21} < 5$

20

19. Summing the given equations & dividing by 4 gives
$v+w+x+y+z = 19$

$\Rightarrow x=4,\ v=6,\ z=4,\ y=8,\ w=3$

-2304

20. $(\frac{1}{2}-x).08 + x(1.00) = \frac{1}{2}(.12)$

$.92x = .02$

$x = \frac{1}{46}$

21. $152\overline{)41201}$ 255

314

870

781

781

22. Consider two rings of 19 people with 19 handshakes around the circumference of each ring. Each person in Ring A can then shake hands with a different person in Ring B, for another 19. $19+19+19 = 57$

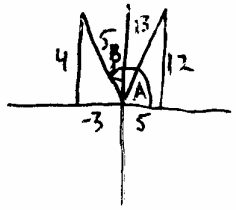
23. $\frac{a(1-r^3)}{1-r} = 30$

$\frac{a(1-r^6)}{1-r} = 270$

divide: $1+r^3 = 9$

$r = 2$

24.



$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$= \frac{\frac{12}{5} + \frac{-4}{3}}{1 - \frac{12}{5}\cdot\frac{-4}{3}} = \frac{16}{63}$

25. 221 0's $\Rightarrow$ 221 5's in the

Prime Factorization $\Rightarrow$ $890 \le N < 895$

890, 893 $\not\equiv 2 \bmod 3$

26. S = 1 because it's the "new" digit

A = 5 because it makes a "new" digit with carrying

C = Ø similarly

$$\begin{array}{r} 55Ø1 \\ +\ \ T0\ 0 \\ \hline 1Ø00\ T \end{array}$$

T = 4, O = 3

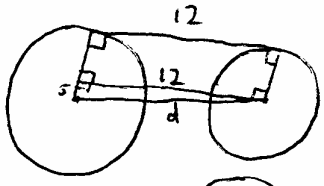
27. $A = XXXXX1$

$\Rightarrow B$ ends in 1, 3, 7, or 9

$10^{5/4} < B < 10^{6/4}$

$B < 10\sqrt{10} \approx 31.9$

28.



$d = \sqrt{5^2+12^2} = 13$

29. $360 = 2^3 \cdot 3^2 \cdot 5$

$= 3 \cdot 4 \cdot 5 \cdot 6$

$2 - 3 - 4 - 5 - 6$

(2)

30. $n = 2^a \cdot 3^b$

$a = 3q+1 \quad b = 2r$

$a = 2s \quad b = 2t-1$

$a = 4, \quad b = 3$

$16 \cdot 27 = 432$

31. $4(\log_3 x)(\log_9 x) - 6\log_{27} x - 12 = 0$

$4(\log_3 x)(\log_3 x^{\frac{1}{2}}) - 6\log_3 x^{\frac{1}{3}} - 12 = 0$

$y = \log_3 x$

$2y^2 - 2y - 12 = 0$

$y^2 - y - 6 = 0$

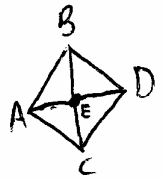
$(y-3)(y+2) = 0$

$y = 3, \quad y = -2$

$\log_3 x = 3, \quad \log_3 x = -2$

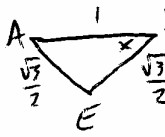
$x = 27, \quad x = \frac{1}{9} \qquad \Sigma = \frac{244}{9}$

32.



$AD = 1$

$AE = AD = \frac{\sqrt{3}}{2}$



$\frac{3}{4} = 1 + \frac{3}{4} - \sqrt{3}\cos x$

$\cos x = \frac{\sqrt{3}}{3}$

33. C = cabin cost

P = per-person cost

N = number of people

$C = NP = (N-1)(P+5) = (N+2)(P-4)$

$5N - P = 5$

$-4N + 2P = 8$

$6N = 18$

$N = 3$

$P = 10$

$\Rightarrow C = 30$

34. $2-2\sin^2(2x)+3\sin(2x)=0$

$y=\sin(2x)$

$2y^2-3y-2=0$

$(2y+1)(y-2)=0$

$y=-\frac{1}{2}$ ~~$y=2$~~

$\sin(2x)=-\frac{1}{2}$

$2x=210°, 330°, 570°, 690°$

$x=105\ \ 165\ \ 285\ \ 345$

$\Sigma=700°=\frac{900}{180}\pi=\frac{45}{9}\pi=\boxed{5\pi}$

35. $9=9^1=3\cdot3$

$2^2\cdot3^2=36$

~~$2^2\cdot5^2=100$~~

~~$3^2\cdot5^2=225$~~

$10=2\cdot5$

$2^4\cdot3=48$

~~$2^4\cdot5=80$~~

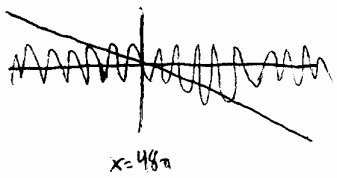
$12=2\cdot6 \quad =3\cdot4 \quad =2\cdot2\cdot3$

~~$2^5\cdot3=96$~~ ~~$2^3\cdot3^2=72$~~ $2^2\cdot3\cdot5=60$

$16=4\cdot2\cdot2$

~~$2^3\cdot3\cdot5=120$~~

$36+48+60=\boxed{144}$

36. 

$x=48\pi$

$0\le x<48\pi: \quad 1+2(23)=47$

$x=48\pi \qquad 1$

$48\pi<x\le96\pi \qquad 47$

$\boxed{95}$

37. $2\left[\frac{32\cdot33\cdot65-4\cdot5\cdot9}{6}\right]$

$-3\left[\frac{32\cdot33-4\cdot5}{2}\right]$

$-4[32-4]$

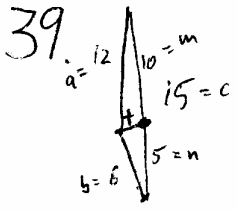
$=2\cdot11410-3\cdot513-4\cdot28$

$=\boxed{21154}$

38. $\triangle AFB$ similar to $\triangle EFD$ w/ ratio 2

$\Rightarrow$ height $AB\to F=\frac{2}{3}AD$

$\frac{A(\triangle AFB)}{A(\square ABCD)}=\frac{\frac{1}{2}\left(\frac{2}{3}AD\right)AB}{AD\cdot AB}=\boxed{\frac{1}{3}}$

39. 

$a^2n+b^2m=t^2c+mcn$

$720+360=15t^2+750$

$330=15t^2$

$22=t^2$

$t=\boxed{\sqrt{22}}$

40. $2, 3, 5, 7, 11$

$1+2+4+6+2=\frac{15}{36}=\boxed{\frac{5}{12}}$