AREA AND VOLUME - CALCULUS
Mu Alpha Theta National Convention 2003

For all questions, answer E. "NOTA" means none of the above answers is correct.

1) Find the area under $f(x)=\begin{cases}4x-3y=8 & x\geq 5\\ x-2y=-3 & x<5\end{cases}$ on the interval [2,7].

A) 15 B) $\frac{175}{8}$ C) $\frac{245}{12}$ D) 18 E) NOTA

2) Find the area of the region bounded by the curves $f(x)=e^x$ and $g(x)=e^{-x}$ and the line y=0 on the interval $(-\infty,\infty)$.

A) $2e$ B) 1 C) e D) 2 E) NOTA

3) Find the area bounded by one of the leaves of the rose curve $r=5\cos(4\theta)$.

A) $\frac{25\pi}{8}$ B) $\frac{25\pi}{16}$ C) $\frac{5\pi}{2}$ D) $\frac{5\pi}{4}$ E) NOTA

4) What is the volume of the solid formed by the revolution about the x-axis of the region bounded by the curve $f(x)=\frac{1}{x}$, the x-axis, the line x=1, and the line x=e?

A) $\frac{\pi(e-1)}{e}$ B) πe C) $e^{-1}+1$ D) $\frac{\pi}{e^2}+1$ E) NOTA

5) Find the volume of the solid figure that is formed by taking semi-circular cross sections perpendicular to the x-axis along the graph $f(x)=\sin(2x)$ on the interval $\left[0,\frac{\pi}{2}\right]$.

A) $\frac{\pi^2}{16}$ B) $\frac{\pi^2}{32}$ C) $\frac{\pi^2}{4}$ D) $\frac{\pi^2}{8}$ E) NOTA

6) Use the trapezoidal rule with n=4 to approximate $\int_0^4 x\sqrt{1+x^4}\,dx$ to the nearest thousandth.

A) 64.980 B) 63.050 C) 64.992 D) 68.889 E) NOTA

7) Find the area of the region enclosed by the following three functions: $y=-x^2+4x+5$, $9x+2y=36$, and $7x+4y=39$, on the interval where $9x+2y=36>7x+4y=39$

A) $\frac{35}{12}$ B) $\frac{37}{24}$ C) $\frac{26}{9}$ D) $\frac{11}{4}$ E) NOTA

8) What is the volume of the torus formed when $x^2+y^2+10x-6y-2=0$ is revolved about the line x=4?

A) $2916\pi^2$ B) $216\pi^2$ C) $54\pi^2$ D) $324\pi^2$ E) NOTA

9) Find the area of the surface formed by the revolution of the graph $r = f(\theta) = 2\cos\theta$ about the line $\theta = \frac{\pi}{2}$.

A) $2\pi^2$ B) $\sqrt{2}\pi^2$ C) $4\pi^2$ D) 2π E) NOTA

10) Find the volume of the solid formed by revolving the region bounded by $f(x) = e^{2x+1}$, $g(x) = 0$, $x = 0$, and $x = 3$ about the line x=0.

A) $\frac{e\pi\left(e^6 - 1\right)}{2}$ B) $\frac{e\pi\left(5e^6 + 1\right)}{2}$ C) $\frac{\left(5e^7 + e\right)}{4}$ D) $e\pi\left(e^6 - 1\right)$ E) NOTA

11) Which of the following is equal to the volume of the solid formed by revolving the region bounded by $y = \tan x$, y=0, and $x = \frac{\pi}{4}$ about the line y= π?

A) $\pi\int_0^{\frac{\pi}{4}}(\pi - \tan x)^2\,dx + \frac{\pi^3}{8}$ B) $\pi\int_0^{\frac{\pi}{4}}(\tan x)^2\,dx$

C) $-\pi\int_0^{\frac{\pi}{4}}(\pi - \tan x)^2\,dx + \frac{\pi^4}{4}$ D) $\int_0^{\frac{\pi}{4}}\left[\pi^2 - (\pi - \tan x)^2\right]dx$ E) NOTA

12) Given that $\int_a^{-1}[x(x+5)]dx = \frac{77}{6}$, find a.

A) –5 B) –14 C) –10 D) –6 E) NOTA

13) Find the area bounded by $f(x) = 2$, $g(x) = x^{\frac{2}{7}}$, and $x = 0$.

A) $\frac{14\left(\sqrt[7]{4}\right)}{9}$ B) $\frac{112\sqrt{2}}{9}$ C) $\frac{65\sqrt{2}}{9}$ D) $\frac{32\sqrt{2}}{9}$ E) NOTA

14) Use Simpson's Rule with n=6 to approximate $\int_{-\pi}^{\pi} x\sin x\,dx$ to two decimal places.

A) 6.33 B) 6.28 C) 5.70 D) 5.48 E) NOTA

15) What is the expression for the surface area of revolution formed by revolving the graph $f(x) = \arctan x$ about the y-axis on the interval $0 \le x \le \frac{\pi}{4}$?

A) $\pi\int_0^{\frac{\pi}{4}}\arctan x\sqrt{\frac{1}{1+x^2}}dx$ B) $2\pi\int_0^{\frac{\pi}{4}}x\sqrt{\frac{x^4+2x^2+2}{x^4+2x^2+1}}dx$

C) $\pi \int_0^{\arctan\frac{\pi}{4}} \tan y \sqrt{\frac{\cos^4 y+1}{\cos^4 y}}dy$ D) $2\pi \int_0^{\arctan\frac{\pi}{4}} y\sqrt{\frac{y^4+2y^2+2}{y^4+2y^2+1}}dy$ E) NOTA

16) Find the volume of the solid formed by revolving the graph of $y=\frac{1}{x^2+1}$ about the line x=1 on [1, 2].

A) $\frac{\ln 2}{2}-\frac{\pi}{4}$ B) $\pi\left(\frac{\pi}{8}-\frac{\arctan(1/2)}{2}-\frac{1}{20}\right)$

C) $\pi\ln(2)$ D) $2\pi\left(\frac{\ln(5/2)}{2}+\arctan\left(\frac{1}{2}\right)-\frac{\pi}{4}\right)$ E) NOTA

17) Find the volume of the solid formed by revolving the function $f(x)=-x^2+10x-21$ about the x-axis on the interval where x>0, y>0, and $f(x)$ is concave down.

A) $\frac{320\pi}{3}$ B) $\frac{5831\pi}{15}$ C) $\frac{32\pi}{3}$ D) $\frac{1024\pi}{15}$ E) NOTA

18) Find the surface area of the torus formed by revolving the graph $x^2+y^2-8x+12y+27=0$ about the line 5x -12y = -77 .

A) $4225\pi^2$ B) $290\pi^2$ C) $260\pi^2$ D) $420\pi^2$ E) NOTA

19) What is the volume of the solid formed by taking equilateral triangle cross-sections parallel to the x-axis bounded the graph of the curve $x=6+y-y^2$ and the line x=0?

A) $\frac{125\sqrt{3}}{24}$ B) $\frac{625\pi}{6}$ C) $\frac{625\sqrt{3}}{24}$ D) $\frac{625}{24}$ E) NOTA

20) Find the general expression for the volume of a solid formed by revolving the graph of $f(x)=x^{\frac{a}{b}}$ where b>a>0 about the line x=a+b on the interval (0,∞).

A) $2\pi\int_0^{a+b} x^{\frac{a}{b}}(a+b-x)dx$ B) $\pi\int_0^{a+b} x^{\frac{2a}{b}}dx$ C) $\int_0^{a+b} x^{\frac{a}{b}+1}dx$ D) $\int_0^{a+b} x^{\frac{a}{b}}(x-a-b)dx$ E) NOTA

21) What is the volume of the figure formed by revolving the region bounded by the functions $f(x)=x^2-1$ and $g(x)=x+1$ about the line y=x+8?

A) $\frac{56169\pi}{400}$ B) $\frac{711\pi\sqrt{2}}{20}$ C) $\frac{2151\pi\sqrt{2}}{80}$ D) $\frac{514089\pi}{6400}$ E) NOTA

22) Find the volume of the solid formed by revolving the graph of $f(x)=\ln x$ about the line x=0 on the interval [1, e].

A) 2π B) $\frac{\pi\left(e^2+1\right)}{2}$ C) $\pi\left(e-2\right)$ D) $\frac{\pi\left(e^2+1\right)}{4}$ E) NOTA

23) What is the approximate area using 4 right hand rectangles of $\int_0^1 \tan x dx$ to three decimal places?

A) .616 B) .433 C) .610 D) .823 E) NOTA

24) Find the area inside the outer loop and outside the inner loop of the limaçon r=1-2 cos θ.

A) $\frac{3\sqrt{3}+4\pi}{2}$ B) $2\sqrt{3}+\frac{4\pi}{3}$ C) $3\sqrt{3}+4\pi$ D) $\frac{\pi-\sqrt{3}}{2}$ E) NOTA

25) What is the area bounded by the functions $f(x)=\left|3x-4\right|$ and $g(x)=\left|x+3\right|$?

A) $\frac{129}{16}$ B) $\frac{169}{8}$ C) $\frac{169}{24}$ D) $\frac{169}{16}$ E) NOTA

26) What is the volume of the solid formed when the region bounded by $\phi(x)=\frac{1}{\sqrt{x}}$ and y=0 is rotated about the line $x=-1$ on the interval [1,4]?

A) $\frac{20\pi}{3}$ B) $\frac{80\pi}{3}$ C) $\frac{16\pi}{3}$ D) $\frac{28\pi}{3}$ E) NOTA

27) Find the volume of the solid formed by revolving the region bounded by $y=\sqrt{\arcsin x}$, and y=0 about the line y=0 on the interval [0,1].

A) $\frac{\pi^2}{4}$ B) $\frac{\pi}{2}-1$ C) $\pi^2-2\pi$ D) $\frac{\pi\left(\pi-2\right)}{2}$ E) NOTA

28) Find the volume of the solid formed by taking isosceles right triangle cross-sections perpendicular to the x-axis, with the hypotenuse of the cross-section triangles being in the xy-plane, along the graph of $y=4x^2$ on the interval [0,1].

A) $\frac{8}{5}$ B) $\frac{32}{5}$ C) $\frac{4}{5}$ D) $\frac{4\sqrt{3}}{5}$ E) NOTA

29) Find the volume of the solid formed when the region bounded by $y=\frac{1}{\sqrt{x^2-7x+12}}$, y=0, x=0, and x=2 is revolved about the line y=0.

A) $2\pi\ln\left(\frac{27}{16}\right)$ B) $\pi\ln\left(\frac{3}{2}\right)$ C) $\frac{\pi}{2}\ln(3)$ D) $\frac{\pi}{2}\ln(16)$ E) NOTA

30) What is the area of the region bounded by y=sin x and y=cos x and x=0 for x>0?

A) $1-\sqrt{2}$ B) 1 C) $\sqrt{2}-1$ D) $\sqrt{3}+1$ E) NOTA