2003 Mu Alpha Theta National Convention
Calculus Applications Topic Test – Mu Division

Notes: All functions and numbers in this test take on real number values.

1. Find the derivative of $f(x) = (x+1)(x+2)(x+3)$ at $x = 1$.

   (A) 0 (B) 13 (C) 26 (D) 39 (E) NOTA

2. Evaluate: $\lim_{x \to 0} \dfrac{\sin(5x)}{3x}$

   (A) 2 (B) $\dfrac{5}{3}$ (C) $\dfrac{4}{3}$ (D) 1 (E) NOTA

3. Find the average value of e^x as x increases from 1 to 3.

   (A) $\dfrac{e^3 - e}{2}$ (B) e (C) $2e$ (D) e^3 (E) NOTA

4. Let $f(x) = 3x^2$. Evaluate: $\lim_{h \to 0} \dfrac{f(2+h) - f(2)}{h}$

   (A) 15 (B) 14 (C) 13 (D) 12 (E) NOTA

5. Which of the following statements are always true?

   (A) If $f(x)$ is continuous everywhere, it is also differentiable everywhere.
   (B) If $f(x)$ is differentiable at $x = a$, it is also continuous there.
   (C) There exists a differentiable function that's not continuous.
   (D) There does not exist a continuous function that's not differentiable.
   (E) NOTA

6. Let $g(x) = \sin x$. Find the 2003$^{\text{rd}}$ derivative of g with respect to x.

   (A) $\cos x$ (B) $-\sin x$ (C) $-\cos x$ (D) $\sin x$ (E) NOTA

7. If x is a real number, find the maximum value of $\sin x + \cos x$.

   (A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) 2 (D) $\sqrt{5}$ (E) NOTA

8. Find y' if $y = x^3 \cos(4x)$.

   (A) $3x^2 \cos(4x) - 4x^3 \sin(4x)$ (B) $12x^2 \sin(4x)$

   (C) $3x^2 \sin(4x) - 4x^3 \cos(4x)$ (D) $3x^2 \cos(4x) + 4x^3 \sin(4x)$ (E) NOTA

9. Find the derivative of $h(x) = \sin^6 x + \cos^6 x + 3 \sin x \cos x (\sin^2 x + \cos^2 x)$ at $x = \pi/4$.

   (A) $\dfrac{\sqrt{2}}{2}$ (B) $\sqrt{3}$ (C) 0 (D) 1 (E) NOTA

10. Evaluate $\dfrac{dy}{dx}$ at $(2003, 2003)$ if $x^3 + y^3 = axy$, where a is a constant not equal to 6009.

(A) 1 (B) -1 (C) 2 (D) -2 (E) NOTA

11. If $x(t) = (t^2 + 3t + 1)e^{2\ln t^2}$ describes the position of an antiparticle moving along the x-axis, find the acceleration of the antiparticle when $t = 1$.

(A) 5 (B) 25 (C) 61 (D) 102 (E) NOTA

12. If two resistors with resistances R_1 and R_2 (in ohms) are connected in parallel, then the total resistance R, measured in ohms, is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

If R_1 and R_2 are increasing at rates of 0.4 ohm per second and 0.5 ohm per second, respectively, how fast is R changing in ohms per second when $R_1 = 100$ ohms and $R_2 = 200$ ohms?

(A) $\dfrac{7}{30}$ (B) $\dfrac{11}{18}$ (C) $\dfrac{8}{45}$ (D) $\dfrac{4}{9}$ (E) NOTA

13. A ladder 15 feet long rests against a vertical wall perpendicular to the ground. If the bottom of the ladder slides away from the wall at a speed of 3 feet per second, how fast is the angle between the top of the ladder and the wall changing in feet per second at the moment the angle is $\pi/4$ radian?

(A) $\dfrac{\sqrt{2}}{5}$ (B) 2 (C) $\dfrac{\sqrt{3}}{5}$ (D) $\dfrac{2}{5}$ (E) NOTA

14. Let $A(c)$ equal the area of the region bounded by the graph of $f(x) = x^4 + 2x^2 + 12$ and the x-axis from $x = 0$ to $x = c$, where $c \leq 100$. What is the value of $A'(5)$?

(A) 680 (B) 685 (C) 687 (D) 690 (E) NOTA

15. A telescope is sold at a price of $\displaystyle\sum_{x=1}^{2003} \frac{1}{x(x+1)}$ dollars. How much does the telescope cost, in dollars?

(A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

16. Let a_n be a sequence given by $a_1 = 10^{-2003}$ and $a_{n+1}(a_n + 1) = a_n$ for $n \geq 1$. Which of the following is an upper bound for $\displaystyle\sum_{n=1}^{\infty} a_n$?

(A) 1 (B) $\dfrac{\pi}{2}$ (C) $\sqrt{e}$ (D) $5\ln 10$ (E) NOTA

17. Evaluate: $\lim_{x \to \pi/2} \dfrac{e^{\cos x} - 1}{x - \pi/2}$

(A) 0 (B) -1 (C) 1 (D) $+\infty$ (E) NOTA

18. Which of the following intervals does $f(x) = x^3 - 26x + 1$ have a root?

(A) $0 < x < 5$ (B) $6 < x < 11$

(C) $12 < x < 17$ (D) $18 < x < 23$ (E) NOTA

19. Find the area of largest rectangle that can be inscribed in the graph of $y = -4x^2 + 64$ (where $y \geq 0$) and the x-axis given that one side of the rectangle lies on the x-axis.

(A) $\dfrac{2^7}{\sqrt{3}}$ (B) $\dfrac{2^8}{3}$ (C) $\dfrac{2^9}{27}$ (D) $\dfrac{2^{10}\sqrt{3}}{9}$ (E) NOTA

20. What is the maximum area of triangle ABC, given that $A = (0, 0)$, $B = (0, 10)$, and C is a point on the curve given by $y = x^3 + 1$, where $|x| \leq 20$?

(A) 60 (B) 80 (C) 100 (D) 120 (E) NOTA

21. Suppose that $y = f(x)$ is a function such that $f'(x) = y$, and $f(5) = 3$. Find, to the nearest hundredth, the value of $f(6)$.

(A) 3 (B) 8.15 (C) 9.74 (D) 0 (E) NOTA

22. Find the volume of the solid obtained by rotating the region between the graphs of $y = x$ and $y = x^3$ about the y-axis.

(A) $\dfrac{\pi}{15}$ (B) $\dfrac{2\pi}{15}$ (C) $\dfrac{4\pi}{15}$ (D) $\dfrac{8\pi}{15}$ (E) NOTA

23. Find the point where the curves $y = x^3 - 3x + 4$ and $y = 3(x^2 - x)$ are tangent to each other.

(A) $(2, 6)$ (B) $(-1, 9)$ (C) $(-1, 6)$ (D) $(2, 9)$ (E) NOTA

24. What is the volume of solid formed when the circle given by $(x - 6)^2 + y^2 = 1$ is revolved around the y-axis?

(A) $10\pi^2$ (B) $12\pi^2$ (C) $14\pi^2$ (D) $36\pi^2$ (E) NOTA

25. What is the area of an ellipse with major axis 10 and minor axis 6?

(A) 15π (B) 30π (C) 60π (D) 120π (E) NOTA

26. Find the area in-between the graphs of $y = x^2$ and $y = 10x$.

(A) 125 (B) $\dfrac{500}{3}$ (C) 250 (D) 500 (E) NOTA

27. Solid A is 10 meters tall. The area of the cross sections of A parallel to the ground is given by $C(x) = 3x^2 + 3$, where x is the distance from the cross section to the ground in meters. What is the volume of A, in cubic meters?

(A) 1000 (B) 1010 (C) 1020 (D) 1030 (E) NOTA

28. Evaluate: $\lim_{x\to\infty} \left(\frac{2^x - 1}{2}\right)^{1/x}$

(A) 1 (B) 2 (C) e (D) e^2 (E) NOTA

29. How many polynomials $P(x)$ with integer coefficients exist such that $P'(x) > 0$ and $(P(x))^2 + 4 \leq 4P(x^2)$ for all x?

(A) 0 (B) 1 (C) 2 (D) 3 (E) NOTA

30. In the xy-plane, the points $(0, 0)$, $(2, 0)$, and $(0, \sqrt{3})$ are labelled A, B, and C in some order. Find the sum of the coordinates of the point P on the same plane inside triangle ABC such that the sum of the distances from P to each of A, B, and C is as small as possible.

(A) $\frac{5 + 3\sqrt{3}}{13}$ (B) $\frac{1}{2} + \sqrt{\frac{5}{3}}$ (C) $1 + \frac{\sqrt{3}}{2}$ (D) $\frac{\sqrt{7} - 2}{6}$ (E) NOTA