1.1. $\frac{(10t+u)-(t+u)}{9}$

$= \frac{9t}{9} = (+)$

1.2. $220-100=120$

$340-220=120$

For 120 to be the difference b/t squares, it must be the product of two odd numbers (it's not) or of two even numbers, as it must be the sum of consecutive odd numbers.

$120 = 2\cdot2\cdot30 = (2\cdot2)\cdot(2\cdot3\cdot5)$

$= 2\cdot60 \Rightarrow 59, 61 \Rightarrow 29^2, 31^2$

$= 4\cdot30 \Rightarrow 27, 29, 31, 33 \Rightarrow 13^2, 17^2$

$= 6\cdot20 \Rightarrow 15, 17, 19, 21, 23, 25 \Rightarrow 7^2, 13^2$

$= 10\cdot12 \Rightarrow 3, 5, 7, 9, 11, 13, 15, 17, 19, 21 \Rightarrow 1^2, 11^2$

The three squares are 49, 169 & 289, so the number to add is (-51)

1.3. $216-208=8$

$208-200=8$

8 is small enough there should only be one way to achieve this...

$-8, 0, 8 \Rightarrow (-208)$

1.4. Make a table of possible distances.

	0	1	2	3
1	~~1~~	$\sqrt{2}$	~~$\sqrt{5}$~~	~~$\sqrt{10}$~~
2	$\sqrt{4}$	$\sqrt{5}$	$\sqrt{8}$	~~$\sqrt{13}$~~
3	$\sqrt{9}$	$\sqrt{10}$	$\sqrt{13}$	$\sqrt{18}$

There are 9 possible unique distances, & 9 possible pairs of consecutive numbers, so there's a 1:1 correlation. Start at the largest separation (fewest options) & work down.

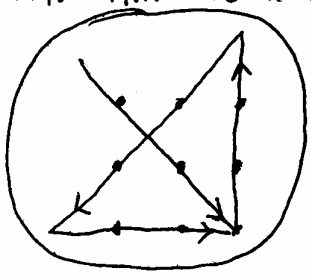
20 & 19 must be in opposite corners (2 ways to do this), 18 must be next to 20, 17 needs to be 3×1 away from 18, which can only happen if 20 starts in the upper left putting it next to 19. 16 is next to 18, 15 must be 2×2 away from 16, but that's only possible for one of our options, giving

19	17		
			15
	16	18	20

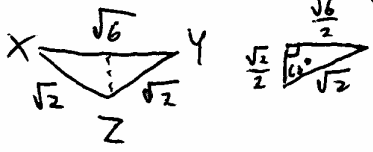
The rest is forced.

2.1. The trick is to think spatially, not in a plane, which lets you think of a tetrahedron.

2.2. You just need to play around with this one a bit.



2.3. a. XYZ is a $\sqrt{2}, \sqrt{2}, \sqrt{2}$ triangle, Equilateral $\Rightarrow 60^\circ = \frac{\pi}{3}$

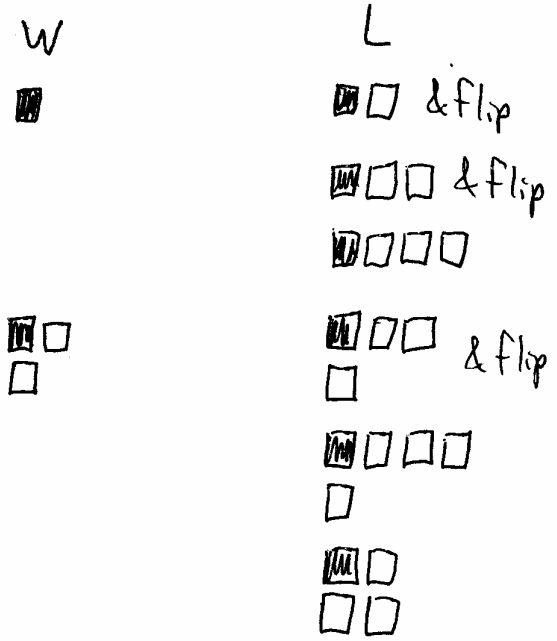
b. 

You can break it into two 60° angles $\Rightarrow 120^\circ = \frac{2\pi}{3}$

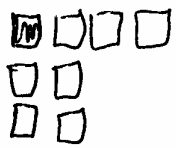
c. angle XZY won't change if you move Y $\sqrt{2}$ units closer to Z along the same line. Once you've done that, XYZ is a $2\sqrt{2}, 2\sqrt{2}, 2\sqrt{2}$ triangle $\Rightarrow 60^\circ = \frac{\pi}{3}$

2.3d. XYZ is a $3\sqrt{2}, 3\sqrt{2}, 6$ triangle, $\Rightarrow 45^\circ = \frac{\pi}{4}$

3. The position a player leaves the crackers in ~~determines~~ at the end of his/her turn determines whether they will win (assuming the other player plays optimally).



Working backwards, you can show that you should go first & ~~not~~ leave



4.1.a. In the 1-2-3 game, the trick is to offer your opponent $4n+1$ matches. Whether he removes 1, 2, or 3 matches, you can remove 3, 2, or 1 match to once again offer $4n+1$ matches. Eventually, you're offering 1 match, and your opponent loses. So, you should go first & take 1 match. After that, 70 turns take place, & your opponent is left with one match to take on his last turn. $70+1+1=72$.

4.1.b. This game is best worked backwards. Clearly, if there's just one match to take, you lose. This means that 2, 5, 10, 17, 26, 37, 50, 65, 82, 101, & 122 are all winning positions. 3 is a losing position, so 4, 7, 12, 19, 28, 39, 52, 67, 84, 103, & 124 are winning positions.

6 is a losing position, so 7, 10, 15, 22, 31, 42, 55, 70, 87, 106, & 127 are winning positions.

You can show that 21 is a losing position, so 142 is a winning one. You'll start & take 121. He'll take 1, you'll take 9, he'll take 4, you'll take 4, he'll take 1, you'll take 1, & he'll take 1, for a total of 8 turns.

4.2. List possible squares:

1
4
9
16
25
36
49
64
81
100
121
144
169
196

a. $64+144$ is the only 2-level solution.

b. [illegible]

There are 3 6-tier solutions.

1, 9, 16, 25, 36, 121

1, 4, 9, 49, 64, 81

1, 16, 25, 36, 49, 81

5.1. There are 525960 minutes per year, so

A. 525,960 → 525,600,000 / year

B. 52,596,000 → 105,192,000

C. → 157,788,000

D. → 210,384,000

If you assume that it takes between ten & 20 years for the population to go up 10^9, (B) is the obvious answer. Online research confirms this.

5.2. Sometimes (2000 for example) we don't have a leap day when we "should", so a year must be less than 365.25 days. (A)

5.3. The Earth rotates the same direction it orbits the sun. There are probably several ways to intuit this; I use stellar formation theories. Noon occurs once every 24 hours, but the earth must rotate a little bit "extra" to make that happen. (A)

5.4. For a knight, this would be a false statement. For a knave it would be true. (C)

5.5. In a knight/knave marriage, both would be telling the truth. (A)

5.6.

	S	M	T	W	Th	F	S
Lion	T	L	L	L	T	T	T
Unicorn	T	T	T	T	L	L	L

They never lie on the same day, so one is telling the truth & the other is lying. The W-Th switch is the crucial point. (E)

5.7. There is a total of 2 liters of stuff, which ends up 1 liter in each jar, so whatever quantity went one way had to have gone the other. (C)

5.8. Inhabitants could have from 0 → 85123 grey hairs, ⇒ 85124 people (D)

6. Typically these are solved by filling in a table:

		Red	Blue	White	Black	P_1	P_2	P_3	P_4
W_3	CP75	X_3	X_3	X	☆	$☆_3$	X_3	X_2	X_3
W	MM+	X	X	☆	X	X_2	X_2	$☆_2$	X_2
W	S5000	☆	X	X	X	X_3	X_3	X_2	$☆_3$
DW	SSII	X	☆	X	X	X_3	$☆_3$	X_2	X_3
	9_3 P_1	X_4	X	X_5	☆				
	11_3 P_2	X_4	☆	X	X				
W	12_2 P_3	X_4	X	☆	X				
W_1	13_3 P_4	$☆_4$	X_4	X_1	X_1				

W W
9_3 11_3 12_2 13_3

7. $A = 1234 \cdot 10001 \cdot 1313 \cdot 10001$

$B = 1212 \cdot 10001 \cdot 1231 \cdot 10001$

$C = 1111 \cdot 10001 \cdot 1010 \cdot 10001$

$D = 12345678 \cdot 1011 \cdot 10001$

$A+B+C = 10001^2 \cdot (1620242 + 1491972 + 1122110)$

$= 10001^2 \cdot 4234324$

$A+B+C = 10001 \cdot 42347474324$

$D = 10001 \cdot 12481480458$

$\Rightarrow A+B+C+D = 10001 \cdot 54828954782$

$= 548,344,376,774,782$

8.1. You either see it, or you don't.

8.2. There are many, many words you can make. My favorite is "chimera".

9. If you re-assemble it, it's fairly easy to read "What is the sum of seven and eight?"