Mu Alpha Theta National Convention: Denver, 2001

Individual Test – Euclidean Division

1. What is 10% of 16% of 3200?

   (A) 48.8 (B) 51.2 (C) 2419.2 (D) 62.4 (E) NOTA

2. Solve for x: $3x + 2 - 9 - 6x = 9x - (2x + 1) + 6$

   (A) $-\frac{6}{5}$ (B) $-\frac{11}{10}$ (C) -1 (D) $-\frac{9}{10}$ (E) NOTA

3. If $\frac{x}{y} = \frac{4}{7}$ and $\frac{y}{z} = \frac{14}{3}$, evaluate $\frac{x+z}{z}$.

   (A) $\frac{8}{3}$ (B) $\frac{55}{49}$ (C) $\frac{11}{3}$ (D) $\frac{6}{49}$ (E) NOTA

4. Line l intersects sides $\overline{AB}$ and $\overline{CD}$ of rhombus ABCD at E and F, respectively. Given $\angle AEF = 83°$, determine the measure of $\angle EFC$.

   (A) 97° (B) 7° (C) 83° (D) 106° (E) NOTA

5. Find the smallest natural number evenly divisible by 42, 175, and 392.

   (A) 210 (B) 13720 (C) 29400 (D) 68600 (E) NOTA

6. Evaluate: $1 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{4}}}$

   (A) $\frac{8}{5}$ (B) $\frac{43}{30}$ (C) $\frac{23}{7}$ (D) $\frac{25}{21}$ (E) NOTA

7. What is the diameter of a circle whose area is 49π?

   (A) $\frac{7}{\sqrt{2}}$ (B) 7 (C) 14 (D) $7\sqrt{2}$ (E) NOTA

8. Compute the value of $\frac{16x^7y^9}{24xy^3} \div \frac{8x^6y^6}{9x^2y^2}$ when $x = 3$ and $y = 4$.

   (A) $3^7 \cdot 2^{24}$ (B) 27 (C) $85.\overline{3}$ (D) 108 (E) NOTA

9. In the realm of celebrity dining, it is considered polite to give the waiter a tip worth 12% of the cost of the meal. If an actress leaves a $432 tip, how much did the meal cost?

(A) $3200 (B) $3400 (C) $3600 (D) $3800 (E) NOTA

10. Which of the following is a number that, when added to its reciprocal, produces $\frac{10}{3}$?

(A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA

11. Due to a slanted ceiling, the vertical wall at one end of an attic has the shape of a trapezoid with two vertical sides perpendicular to the horizontal floor. The wall is 8 ft. high at one end, 10 ft. wide, and only 3 ft. high at the other end. How many square feet of wallpaper would be needed to cover this wall?

(A) 240 ft² (B) 120 ft² (C) 110 ft² (D) 55 ft² (E) NOTA

12. What is the prime factorization of 5292?

(A) $2^2 \cdot 3^2 \cdot 7^3$ (B) $2^2 \cdot 3^3 \cdot 7^2$ (C) $2^3 \cdot 3^2 \cdot 7$ (D) $2^3 \cdot 3^2 \cdot 7^2$ (E) NOTA

13. The arithmetic mean of *a* and *b* is 8 while the arithmetic mean of *c* and *d* is 44. What is the arithmetic mean of *a*, *b*, *c*, and *d*?

(A) 7.5 (B) 13 (C) 26 (D) 52 (E) NOTA

14. Find the solution set for x: $3x - 2 \le -8x + 7$

(A) $x \le \frac{8}{5}$ (B) $x \le -\frac{5}{11}$ (C) $x \ge -\frac{9}{5}$ (D) $x \le \frac{9}{11}$ (E) NOTA

15. How many counting numbers less than 56998 are multiples of 3 or 2?

(A) 56997 (B) 20898 (C) 47498 (D) 37998 (E) NOTA

16. Find the mode of the following data set: 5, -3, 7, 14, 5, 8, 2, -4, 6, 5, 6, 19

(A) 5 (B) 23 (C) 5.5 (D) $\frac{35}{6}$ (E) NOTA

17. Since Richard's newly purchased home was so large, the seller provided him with a map where $1\frac{1}{2}$ inches represents 25 feet. If two bathrooms are $6\frac{3}{8}$ inches apart on the map, how far apart are the bathrooms?

(A) 103.125 ft (B) 100 ft (C) 105.5 ft (D) 106.25 ft (E) NOTA

18. What is the perimeter of a rectangle with area 252 and width 12?

(A) 34 (B) 66 (C) 33 (D) 68 (E) NOTA

19. The line with equation $y = 2x + 6$ never passes through which quadrant?

(A) IV (B) III (C) II (D) I (E) NOTA

20. How many diagonals can be drawn in a 23-sided polygon?

(A) 153 (B) 253 (C) 161 (D) 230 (E) NOTA

21. Simplify: $\dfrac{1-\dfrac{a}{b}}{\dfrac{b}{a}-1}$

(A) -1 (B) $\frac{a}{b}$ (C) 1 (D) $\frac{b}{a}$ (E) NOTA

22. The Fibonacci numbers are defined as follows: The first two numbers of the sequence are 1 and from the third term onward, each term is the sum of the previous two terms. If the 55^{th} Fibonacci number is x and the 56^{th} Fibonacci number is y, which of the following is an expression for the 51^{st} Fibonacci number in terms of x and y?

(A) $5y - 2x$ (B) $3y - 5x$ (C) $2y - 3x$ (D) $5x - 3y$ (E) NOTA

23. Find the equation of the line passing through (3, 6) and (8, -1) in slope-intercept form.

(A) $5y + 7x = 57$ (B) $y = \frac{7}{5}x + \frac{61}{5}$

(C) $y = -\frac{7}{5}x + \frac{51}{5}$ (D) $5y - 7x = -27$ (E) NOTA

24. The diagonals of a rhombus are 16 and 30. What is its perimeter?

(A) 68 (B) 100 (C) 40 (D) 52 (E) NOTA

25. Solve for x: $10x^2 - 23x - 42 = 0$

(A) $\frac{7}{5}, -3$ (B) -1.3, 3.1 (C) $-\frac{6}{5}, \frac{7}{2}$ (D) 2.3, -4.2 (E) NOTA

26. In ΔABC, AD is drawn to BC, splitting it in a ratio of $\frac{BD}{DC} = \frac{3}{5}$. What is the ratio of the area of ΔABD to ΔABC?

(A) 3:8 (B) 3:5 (C) 5:8 (D) 2:3 (E) NOTA

27. What are the first five terms of the sequence explicitly given by $a_n = 2n^2 - 3$ for $n \geq 1$?

(A) -1, 5, 15, 29, 47 (B) -3, -1, 5, 15, 29
(C) -3, -1, 1, 3, 5 (D) -1, 1, 3, 5, 7 (E) NOTA

28. The sum of two numbers is 52. If the larger is divided by the smaller, the resulting quotient is 9 with a remainder of 2. What is the smaller number?

(A) 8 (B) 7 (C) 6 (D) 5 (E) NOTA

29. For real numbers a and b, define $a \otimes b = \frac{ab}{\sqrt{a^2 + b^2}}$. Find the value of $(3 \otimes 4) + (6 \otimes 8)$.

(A) $\frac{24}{5}$ (B) $\frac{7}{12}$ (C) $\frac{7}{24}$ (D) $\frac{12}{5}$ (E) NOTA

30. Simplify: $\frac{6a^2 - 7a + 2}{10a - 15a^2}$

(A) $\frac{2a+1}{5-3a}$ (B) $\frac{1}{3a-1}$ (C) $\frac{3a+1}{2a}$ (D) $\frac{1-2a}{5a}$ (E) NOTA

31. What is the <u>total</u> surface area of a right circular cylinder with base radius of 3 inches and a height of 4 inches?

(A) 28π in² (B) 30π in² (C) 36π in² (D) 42π in² (E) NOTA

32. If $k = \frac{1}{x^2}$, express $\frac{3}{k} - k^2$ in terms of x.

(A) $3x^4 - x^2$ (B) $3x^2 - x^4$ (C) $\frac{3x^6 - 1}{x^4}$ (D) $\frac{3 - x^6}{x^2}$ (E) NOTA

33. When multiplied out, how many consecutive zeroes appear at the end of $7 \times 3 \times 2 \times 5 \times 11 \times 2 \times 5 \times 5 \times 13 \times 2 \times 2 \times 5$?

(A) 5 (B) 4 (C) 3 (D) 2 (E) NOTA

34. Simplify: $(1 - \sqrt{3})^3$

(A) $10 - 6\sqrt{3}$ (B) $4 - 2\sqrt{3}$ (C) $4 - 9\sqrt{3}$ (D) $28 - 16\sqrt{3}$ (E) NOTA

35. When the variables represent certain parts of a triangle, the equation $ana + bmb = tct + mcn$ is known as **Stewart's Theorem**. Solve this formula for c.

(A) $\frac{a^2n + b^2m - t^2}{mn}$ (B) $\frac{a^2n - b^2m}{t^2 - mn}$
(C) $\frac{a^2n + b^2m}{t^2 + mn}$ (D) $\frac{a^2n + b^2m - mn}{t^2}$ (E) NOTA

36. Let $f(x) = 6x - 7$. There is a function $g(x)$ such that $f(g(x)) = x$. What is $g(x)$?

(A) $\frac{x}{7} - 6$ (B) $\frac{x}{7} - \frac{6}{7}$ (C) $\frac{x}{6} + 6$ (D) $\frac{x}{6} + \frac{7}{6}$ (E) NOTA

37. The perimeter of a regular hexagon is $54\sqrt{3}$. What is its area?

(A) $243\sqrt{3}$ (B) $\frac{729\sqrt{3}}{2}$ (C) $324\sqrt{3}$ (D) $\frac{243\sqrt{3}}{4}$ (E) NOTA

38. Suppose that x, y, and z are positive integers that add up to 284. If $x < 74$ and $152 \geq y$, what is the minimum value of z?

(A) 57 (B) 58 (C) 59 (D) 60 (E) NOTA

39. If $f(x) = 2x^2 - 6x$ and $g(y) = 2y + 9$, find the value of $f(3 + g(2))$.

(A) 416 (B) 260 (C) 13 (D) 169 (E) NOTA

40. If the lines $4ax + 6by = 7$ and $9x + 3y - 1 = 0$ intersect at right angles, find $\frac{a^2}{b^2}$.

(A) $\frac{1}{4}$ (B) 4 (C) 16 (D) $\frac{1}{16}$ (E) NOTA

41. Mike and JJ have a lawn-mowing service. Mike can mow a standard-sized lawn in sixty minutes, while JJ can mow the same lawn in just forty minutes. How many minutes would it take them to mow the same lawn, working together?

(A) 20 minutes (B) 24 minutes (C) 25 minutes (D) 30 minutes (E) NOTA

42. The measure of an exterior angle of a regular polygon with $2x + 6$ sides is x degrees. How many sides does this polygon have?

(A) 9 (B) 12 (C) 15 (D) 18 (E) NOTA

43. Given that $f(x, y) = ax + by$, evaluate $f\left(\frac{de - bf}{ad - bc}, \frac{af - ce}{ad - bc}\right)$.

(A) a (B) b (C) c (D) d (E) NOTA

44. Evaluate: $\sum_{n=1}^{12^2} n$

(A) 5184 (B) 10440 (C) 4176 (D) 650 (E) NOTA

45. What is the area of a triangle with sides 10, 15, and 10?

(A) $\frac{15\sqrt{11}}{2}$ (B) $\frac{75\sqrt{7}}{4}$ (C) $\frac{30\sqrt{5}}{2}$ (D) 50 (E) NOTA

46. Fishbowl A contains four red fish and two blue fish. Fishbowl B contains one red fish and two blue fish. A fish is selected at random from Fishbowl A and transferred to Fishbowl B. What is the probability that a fish then randomly selected from Fishbowl B will be blue?

(A) $\frac{13}{24}$ (B) $\frac{7}{12}$ (C) $\frac{5}{8}$ (D) $\frac{2}{3}$ (E) NOTA

47. Given a triangle *ABC*, a line *l* is drawn parallel to *BC* and passing through *A*. If *B* and *C* remain fixed, and *A* moves along *l*, which of the following features of ΔABC remains constant?

(A) perimeter, but not side lengths (B) area
(C) side lengths (D) angle measures (E) NOTA

48. Determine the sum of an infinite geometric series with first term 12 and common ratio $\frac{5}{6}$.

(A) 24 (B) 36 (C) 60 (D) 72 (E) NOTA

49. In right triangle *ABC*, *AC* = 6, *CB* = 8, and $\overline{CD}$ bisects right angle *C*, where *D* lies on $\overline{AB}$. Find the length of $\overline{AD}$.

(A) $\frac{24}{5}$ (B) $10-4\sqrt{2}$ (C) 5 (D) $\frac{30}{7}$ (E) NOTA

50. The **harmonic mean** of a set of numbers is defined as the reciprocal of the arithmetic mean of the reciprocals of the numbers. If the harmonic mean of *x* and *y* is $\frac{1}{2}$ while the harmonic mean of $3x$ and $5y$ is 2, find the product *xy*.

(A) $\frac{1}{3}$ (B) 4 (C) $\frac{1}{6}$ (D) 3 (E) NOTA