INTEGRATION - CALCULUS
Mu Alpha Theta National Convention 2003

For all questions, answer E. "NOTA" means none of the above answers is correct.

1) Evaluate $\int_{-5}^{1} |3x+5| dx$

A) –6 B) $\frac{82}{3}$ C) 18 D) $\frac{41}{3}$ E) NOTA

2) Find the volume of the solid formed by the revolution about the x-axis of the region bounded by the curve $f(x) = \sqrt{\ln x}$, the line y=0, and the line x= e.

A) 0 B) π C) πe D) πe^2 E) NOTA

3) $\int x\sqrt{4x+3}\,dx$

A) $\frac{(4x+3)^{5/2}}{40} - \frac{(4x+3)^{3/2}}{8} + C$ B) $\frac{(4x+3)^{5/2}}{10} - \frac{(4x+3)^{3/2}}{2} + C$

C) $\frac{1}{6}(4x+3)^{3/2} + C$ D) $\frac{1}{4}(4x+3)^{3/2} + C$ E) NOTA

4) Find the an approximation for the area under the curve $f(x)= x^2+4$ and above the x-axis on the interval [0,1] using the left hand Riemann Sum with 5 equal subdivisions.

A) $\frac{13}{3}$ B) $\frac{111}{25}$ C) $\frac{131}{25}$ D) $\frac{106}{25}$ E) NOTA

5) Find the area bounded by $f(x)= -x^2+3x+5$ and $g(x)= x^2-6x+12$.

A) $\frac{125}{24}$ B) $\frac{16}{3}$ C) $\frac{36}{7}$ D) $\frac{78}{15}$ E) NOTA

6) Uranium (U^{238}) has a half-life of 4,510,000,000 years. How many years will it take for a sample of U^{238} to decay to 85% of its original mass? (rounded to the nearest year)

A) 3,833,500,000 B) 12,343,714,830 C) 1,057,438,294 D) 1,353,000,000 E) NOTA

7) $\int \frac{\tan x}{\cos^2 x} dx$

A) $-\ln(\cos x) + C$ B) $\frac{\tan^2 x}{2} + C$ C) $\sec^2 x + C$ D) $2\sec^2 x + C$ E) NOTA

8) The integral $\int_a^b f'(x)dx = f(b) - f(a)$ represents which of the following?

A) Mean Value Theorem for Integrals B) Rolle's Theorem
C) Fundamental Theorem of Calculus D) Newton's Method E) NOTA

9) $\int \frac{3+x}{x^2+9}dx$

A) $\ln|x^2+9|+C$ B) $\arctan\left(\frac{x}{3}\right)+C$

C) $\arctan\left(\frac{x}{3}\right)+\ln|x^2+9|+C$ D) $\arctan\left(\frac{x+3}{3}\right)+\ln|x^2+9|+C$ E) NOTA

10) Given: $\int_{-3}^{4} f(x)dx = 7$, $\int_{7}^{4} f(x)dx = 3$, $\int_{-3}^{-5} f(x)dx = 4$. Find $\int_{-5}^{7} f(x)dx$

A) 14 B) 11 C) –14 D) 0 E) NOTA

11) Find the general formula for the volume of the region rotated about the line x=a bounded by $y=x^{2/3}$, y=0, and the line x=a, where a>0.

A) $\frac{9a^{\frac{8}{3}}\pi}{40}$ B) $\frac{6a^{\frac{5}{3}}\pi}{5}$ C) $\frac{9a^{\frac{13}{9}}\pi}{13}$ D) $\frac{9a^{\frac{8}{3}}\pi}{20}$ E) NOTA

12) A 45°C sample of metal is taken out of a flame and left in a room with a constant temperature of 30°C. Twenty minutes later, the temperature is 41.5°C. Find the temperature to the nearest tenth of a degree of the metal forty-five minutes from the time it is taken out of the flame. (This object obeys Newton's Law of Cooling)

A) 37.1°C B) 37.9° C) 38.3°C D) 38.6°C E) NOTA

13) $\int f(x)dx = \frac{2e^x - e^x}{e^{2x}}$ Find f(x).

A) $-e^{-x}$ B) e^{-x} C) $-e^{x}$ D) e^{x} E) NOTA

14) Find the average value of the function f(x)= sin x cos x on the closed interval from 0 to $\frac{\pi}{3}$.

A) $\frac{-3\pi}{8}$ B) $\frac{-9}{8\pi}$ C) $\frac{8\pi}{9}$ D) $\frac{3\pi}{8}$ E) NOTA

15) Using Simpson's rule with n=6, find an approximation to 2 decimal places of $\int_{0}^{6} e^{2x}dx$

A) 106850.74 B) 86192.01 C) 81376.90 D) 74915.27 E) NOTA

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16) Identify which definite integral the expression $\lim_{n\to\infty}\sum_{i=1}^{n} 3\left(-2+\frac{5i}{n}\right)^2\left(\frac{5}{n}\right)$ corresponds to.

A) $\int_3^5 \frac{5}{x}dx$ B) $\int_{-2}^3 3x^2dx$ C) $\int_{-2}^5 2x+5dx$ D) $\int_0^3 15xdx$ E) NOTA

17) $\int \frac{x^3+6x^2-9x+9}{x-3}dx$

A) $\frac{x^3}{3}+x^2-3x+C$ B) $\frac{x^3}{3}+\frac{9x^2}{2}+18x+C$

C) $63\ln|x-3|+\frac{x^3}{3}+x^2-3x+C$ D) $63\ln|x-3|+\frac{x^3}{3}+\frac{9x^2}{2}+18x+C$ E) NOTA

18) Find the particular solution of the differential equation $\frac{dy}{dx}=6x^2y$ passing through the point $(1,e)$.

A) $y=\frac{e^{2x^3}}{e}$ B) $y=Ce^{2x^3}$ C) $y=\ln\left(\frac{e^{2x^3}}{e}\right)$ D) $y=\frac{e^{12x}}{e}$ E) NOTA

19) Find the volume of the solid formed by the revolution of the region bounded by $f(x)=x^2$, $g(x)=4-x^2$, and $x=0$ about the line $y=10$.

A) $\frac{176\pi\sqrt{2}}{3}$ B) $\frac{132\pi\sqrt{2}}{3}$ C) $\frac{128\pi\sqrt{2}}{3}$ D) $\frac{160\pi\sqrt{2}}{3}$ E) NOTA

20) Find the integral expression for the length of arc for the function $y=4x^2-6x+3$ on the interval [0,1].

A) $\pi\int_0^1 \sqrt{64x^3-96x^2+37x}dx$ B) $\int_0^1 x\sqrt{64x^2-96x+37}dx$

C) $\int_0^1 \sqrt{64x^2-96x+37}dx$ D) $\pi\int_0^1 x\sqrt{64x^2-96x+37}dx$ E) NOTA

21) Find the general solution of $\left(x^2+y^2\right)dx=8xydy$.

A) $\frac{C}{x^2}=\left(1-7y^2\right)^8$ B) $y=Ce^{\frac{8x}{x^2+y^2}}$ C) $Cx^6=\left(x^2-7y^2\right)^4$ D) $C\frac{y^2}{2}=\frac{2\ln(x)y^2+x^2}{16}$ E) NOTA

22) Find the volume of the shape formed by taking equilateral triangle cross sections parallel to the y-axis along the graph of the ellipse with endpoints along the major and minor axes (0,20) (0,-20) and (15,0) (-15,0), respectively.

A) $2000\sqrt{3}$ B) $8000\sqrt{3}$ C) $\frac{75\pi\sqrt{3}}{2}$ D) $75\pi\sqrt{3}$ E) NOT

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23) $\int \frac{x}{\sqrt{3x^2+9}}dx$

A) $\frac{\sqrt{3x^2+9}}{3}+C$ B) $\frac{\sqrt{3x^2+9}}{6}+C$ C) $\frac{\sqrt{3x^2+9}}{2}+C$ D) $\frac{\sqrt{3x^2+9}}{12}+C$ E) NOTA

24) Using the trapezoidal method with n=4, find the approximate area to 2 decimal places of $\int_0^6 (6x-x^2)dx$.

A) 38.25 B) 33.75 C) 36.00 D) 32.50 E) NOTA

25) $\int \arctan x dx$

A) $\frac{\arctan^2 x}{2}+C$ B) $\arctan x - \frac{\ln(x^2+1)}{2}+C$

C) $x\arctan x + \ln(x^2+1)+C$ D) $x\arctan x - \frac{\ln(x^2+1)}{2}+C$ E) NOTA

26) Find the integral expression for the area of the surface formed by revolving $f(x)=x^2$ on the interval [1,2] about the y-axis.

A) $2\pi\int_1^4 \sqrt{1+\frac{1}{4y}}dy$ B) $2\pi\int_1^4 y\sqrt{1+\frac{1}{4y}}dy$ C) $2\pi\int_1^2 x\sqrt{1+4x^2}dx$ D) $2\pi\int_1^2 x^2\sqrt{1+4x^2}dx$ E) NOTA

27) Find the centroid of the region bounded by the graphs of $f(x)= 11-x^2$ and $g(x)= 2x+3$.

A) $(1, \frac{27}{4})$ B) $(-1, \frac{10}{3})$ C) $(1, \frac{18}{5})$ D) $(-1, \frac{23}{5})$ E) NOTA

28) $\int \frac{\cos\alpha \csc^2\alpha}{\sqrt{\sin\alpha}}d\alpha$

A) $\frac{-2}{3(\sin\alpha)^{5/2}}+C$ B) $\frac{\sqrt{(\sin\alpha)^3}}{3}+C$ C) $\frac{2}{5\cos\alpha\sin^2\alpha}+C$ D) $\frac{2}{3(\cos\alpha)^{5/3}}+C$ E) NOTA

29) Find the area of one leaf of the polar graph $r=3\cos(2\theta)$.

A) $\frac{9\pi}{8}$ B) $\frac{9\pi}{4}$ C) $\frac{3\pi}{2}$ D) 3π E) NOTA

30) An object of mass 5 pounds is dropped of the top of a building. It takes 6 seconds to fall to the ground. Find the height of the building if gravity is assumed to be 32 ft/s^2.

A) 2880 ft. B) 480 ft. C) 576 ft. D) 524 ft. E) NOTA