**LINEAR ALGEBRA – CALCULUS**
**Mu Alpha Theta National Convention 2003**
**For all questions, answer E. NOTA means none of the above answers is correct.**

1. Given the following system of equations, what is the value of $x + y$?

$$4x - 3y + 2z = 6$$
$$3x - \frac{1}{2}y + z = 12$$

A. 6 B. 9 C. 18 D. cannot be determined E. NOTA

2. If $\vec{a} = (5,8,3)$ and $\vec{b} = (2,-1,4)$, what is the cosine of the acute angle formed between $\vec{a}$ and $\vec{b}$?

A. $\frac{1}{147}$ B. $\frac{2\sqrt{119}}{17}$ C. $\frac{\sqrt{42}}{21}$ D. $\frac{\sqrt{399}}{21}$ E. NOTA

3. Consider the matrix B = $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 5 & 8 & 9 \end{bmatrix}$. Find $|B|$.

A. –2 B. 2 C. 4 D. 8 E. NOTA

4. How many of the following statements concerning the determinants of $n \times n$ matrices $A$, $B$, and $C$ are true?

I) If all the entries of just one row of $A$ are multiplied by $r$, then the determinant of the new matrix is equal to $r^2 \det(A)$.
II) The interchange of two rows of matrix A changes the sign of the determinant of A.
III) If matrices $A$ and $B$ differ only in row $i$, then $\det(A) + \det(B) = \det(C)$ where $C$ differs from $A$ or $B$ only in row $i$, and has as its $i$ th row the sum of the $i$ th rows of $A$ and $B$.
IV) If two rows of $A$ are identical, then $\det(A) = 0$.

A. 1 B. 2 C. 3 D. 4 E. NOTA

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5. If $\vec{a} = (3,-4,-1)$ and $\vec{b} = (0,6,2)$, what is $\vec{a} \times \vec{b}$?

A. (-2,-6,18) B. (-14,6,18) C. (-8,6,12) D. (-2,0,12) E. NOTA

6. How many of the following matrices are in reduced row echelon form?

$$A = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \end{bmatrix} \qquad E = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A. 1 B. 2 C. 3 D. 4 E. NOTA

7. What is the shortest distance between the planes $x + 2y + 2z = 5$ and $x + 2y + 2z = 10$?

A. $\frac{5}{9}$ B. $\frac{5}{4}$ C. $\frac{5}{3}$ D. 5 E. NOTA

8. What is the projection of $\vec{a} = (-3,1,4)$ onto $\vec{b} = (2,-3,1)$?

A. (-10,15,-5) B. $\frac{1}{14}$(-10,15,-5) C. (15,-5,-20) D. $\frac{1}{26}$(15,-5,-20) E. NOTA

9. What is the dimension of the column space of the following matrix?

$$A = \begin{bmatrix} 1 & 3 & 3 & 2 \\ 2 & 6 & 9 & 5 \\ -1 & -3 & 3 & 0 \end{bmatrix}$$

A. 1 B. 2 C. 3 D. 4 E. NOTA

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10. Let $P_3$ be the space of polynomials of degree 3, and let its four basis vectors be $p_1 = 1$, $p_2 = t$, $p_3 = t^2$, and $p_4 = t^3$. For example, if a certain polynomial is $at^3 + bt^2 + ct + d$, then it can be represented as $\vec{p} = (d, c, b, a)$. What is the differentiation matrix $A$ for this basis? (meaning $\frac{d\vec{p}}{dt} = A\vec{p}$)

A. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ B. $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ C. $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix}$ D. $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$ E. NOTA

11. If $A$ is a $5 \times 6$ matrix, $B$ is a $5 \times 7$ matrix, and $C$ is a $7 \times 1$ matrix, what are the dimensions of $A^{\mathrm{T}}BC$?

A. $5 \times 1$ B. $6 \times 1$ C. $6 \times 7$ D. cannot be determined E. NOTA

12. If $\vec{a} = (6,22,15)$ and $\vec{b} = (4,-5,8)$, and $\vec{c} = (0,2,-13)$, what is $\vec{a} - \vec{b} - \vec{c}$?

A. (2,29,-6) B. (2,25,20) C. (10,15,36) D. (10,19,10) E. NOTA

13. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$, what is the trace of the product $AB$?

A. 30 B. 189 C. 225 D. 2025 E. NOTA

14. What is the sum of the eigenvalues of the following matrix?
$\begin{bmatrix} 4 & 5 & 6 \\ 2 & 8 & 3 \\ 1 & 6 & 2 \end{bmatrix}$

A. 11 B. 14 C. 15 D. 64 E. NOTA

15. What is the product of the eigenvalues of the matrix in problem 14?

A. 11 B. 14 C. 15 D. 64 E. NOTA

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16. If $A$, $B$, and $C$ are all $n \times n$ matrices, which of the following is equivalent to $((AB)^{\mathrm{T}} C)^{\mathrm{T}}$?

A. $C^{\mathrm{T}} B^{\mathrm{T}} A^{\mathrm{T}}$ B. $C^{\mathrm{T}} BA$ C. $C^{\mathrm{T}} AB$ D. $ABC^{\mathrm{T}}$ E. NOTA

17. If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, what is the trace of $A^5$?

A. 276 B. 277 C. 7776 D. 7777 E. NOTA

18. How many of the following conditions are necessary and sufficient tests to say an $n \times n$ matrix $A$ is nonsingular?

I) The rows of $A$ span $\Re^n$.
II) The rows of $A$ are linearly independent.
III) There exists a matrix $A^{-1}$ such that $AA^{-1} = A^{-1}A = \mathrm{I}$.
IV) The determinant of $A$ is not zero.
V) Zero is not an eigenvalue of $A$.

A. 2 B. 3 C. 4 D. 5 E. NOTA

19. What is the determinant of $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 6 & 1 & 2 & 5 & 1 \\ 0 & 4 & 0 & 2 & -2 \end{bmatrix}$?

A. -12 B. 0 C. 4 D. 12 E. NOTA

20. Which of the following lines contains the point (6,2,5) and lies in the direction (-1,6,3)?

A. $\ell(t) = (-t + 7)i + (6t - 4)j + (3t + 2)k$ B. $\ell(t) = (-t + 6)i + (6t + 2)j + (3t - 5)k$
C. $\ell(t) = (7t - 1)i + (-4t + 6)j + (2t + 3)k$ D. $\ell(t) = (6t - 1)i + (2t + 6)j + (5t + 3)k$
E. NOTA

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21. Consider the function $f(t) = t^2 - 4t$ in $C[0,3]$. Find $\|f\|_\infty$.

A. –4 B. –3 C. 2 D. 4 E. NOTA

22. Which of the following is equivalent to $\det((A^T B)^{-1})$?

A. $\det(A)^{-1}\det(B)^{-1}$ B. $-\det(A)^{-1}\det(B)^{-1}$ C. $\det(A)\det(B)^{-1}$
D. $\det(A)^{-1}\det(B)$ E. NOTA

23. Given the original sequence of numbers 1234567, which of the following is an odd permutation of the sequence? (A permutation is a one-to-one mapping of the set onto itself. Example: There are $2! = 2 \bullet 1$ permutations in $S_2$ (12 and 21).)

A. 2157463 B. 4637125 C. 5163274 D. 7243615 E. NOTA

24. Which of the following properties must be true of a linear transformation $T$ for every vector $\vec{u}$ and $\vec{v}$ and every scalar $r$?

I) $T(r\vec{u}) = rT(\vec{u})$
II) $T(\vec{u} + \vec{v}) = T(\vec{u}) + T(\vec{v})$
III) $T(\vec{0}) = \vec{0}$

A. I only B. II only C. I and II only D. I, II, and III E. NOTA

25. If a parallelogram with area 10 undergoes a linear transformation $T : \Re^2 \to \Re^2$ defined by $\vec{y} = A\vec{x}$, where $A = \begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}$, what is the area of the resulting figure?

A. 2 B. 5 C. 30 D. 50 E. NOTA

26. Which of the following tests are necessary and sufficient conditions for the real symmetric matrix $A$ to be positive definite?

I) $x^T Ax > 0$ for all nonzero vectors $x$.
II) All of the eigenvalues of $A$ satisfy $\lambda_i \geq 0$.
III) All of the upper left submatrices of $A$ have non-negative determinants.

A. I only B. II only C. I and III only D. I, II, and III E. NOTA

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27. If the sum of the elements in a matrix $A$ is $4-3i$, what is the sum of the elements in $A^{H}$, $A$'s Hermitian matrix?

A. $4-3i$ B. $4+3i$ C. $\frac{1}{25}(4-3i)$ D. $\frac{1}{25}(4+3i)$ E. NOTA

28. Which of the following matrices rotates a vector $(a,b)$ counter-clockwise through an angle $\theta$?

A. $\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$ B. $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ C. $\begin{bmatrix} -\sin\theta & \cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$

D. $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$ E. NOTA

29. Which of the following vectors is a unit vector?

A. (1,1,1) B. (1,1-1) C. (3,4,5) D. $\left(\frac{2}{7},\frac{3}{7},\frac{6}{7}\right)$ E. NOTA

30. Which of the following properties are always true of similar matrices (matrices that represent the same transformation with respect to different bases)?

I) They have the same eigenvalues.
II) They have the same eigenvectors.

A. I only B. II only C. I and II D. neither I or II E. NOTA