MU DIVISION---PROBABILITY TEST

NATIONAL MU ALPHA THETA
MISSISSIPPI 2002

1. The domain of the normal probability density function $f(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$ is:

   A. $(-\infty, \infty)$ B. $(0, \infty)$ C. $(-\infty, 0)$ D. $[-1, 1]$ E. NOTA

2. Consider the normal probability density function f(x) defined above. Evaluate

   $\int_{-\infty}^{\infty} f(x)\, dx$.

   A. 3 B. 0 C. 2 D. 1 E. NOTA

3. The graoh of the probability density function f(x) defined in #1 above reaches it maximum height at y =____. (Round this value to the nearest tenth)

   A. 0 B. 0.7 C. 0.4 D. 1.0 E. NOTA

4. The probability that a certain event A occurs is given by the following expression:

   $\frac{1}{\int_0^1 (x^3 + 1)\, dx}$. Evaluate this expression.

   A. 5/4 B. 1 C. 0.8 D. ½ E. NOTA

5. Find x>0 such that $\int_0^x (t^3 - 2t + 3)\, dt$ is equal to 24 times the probability of rolling a normal 6-sided die and getting a "five". Round x to the nearest hundredth.

   A. 1.63 B. 0.06 C 1.60 D. no such x exists E. NOTA

6. Given that a point underneath the curve y = sin (x) from x = 0 to x = π is randomly selected. What is the probability that the point lies between $x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$?

   A. $\frac{\sqrt{3}}{2}$ B. $\frac{\sqrt{3}}{4}$ C. $\sqrt{3}$ D. $\frac{1}{2}$ E. NOTA

7. Let p = the probability of rolling two normal 6-sided dice and getting a sum of five. Find the slope of the tangent line to the curve $3x^2 + 2y = 7$ at x = *p.*

   A. –1/6 B. 20/3 C. –2 D. 1/9 E. NOTA

8. Which of the following is a probability vector?

a. *A.* $\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$ B. $\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$ C. $\frac{1}{\sqrt{3}}(1,1,1)$ D. $\left(\frac{1}{2},\frac{1}{3},\frac{1}{4}\right)$ E. NOTA

9. A box contains a variable number of marbles, each of which is colored red or brown. The probability of pulling a brown marble from the box on the *nth* try is $\frac{n}{n+2}$. Find the probability of pulling 3 brown and 1 red marble (in any order) in 4 consecutive draws from the box.

A. 2/9 B. 5/18 C. 0.3 D. 29/90 E 1/3

10. A player tosses two fair coins. He wins $10 if 2 tails occur, $5 if one tail occurs, and $1 if no tails occur. Find his expected earnings.

A. $5.00 B. $5.13 C. $5.25 D. $7.50 E. NOTA

11. A box contains 5 red balls and 7 green ones. Four balls are selected at random without replacement. What is the probability that three of the four selected balls are green?

A: $\frac{3}{35}$ B: $\frac{35}{396}$ C: $\frac{35}{99}$ D: $\frac{5}{7}$ E. NOTA

12. A class consists of 10 students who received an A for the final exam, 15 who received a B, and 20 who received a C. Five students are selected at random from this group. What is the probability that at least one of the five students selected received an A for the final exam.

A: 0.6993 B: 0.2675 C: 0.7343 D: 0.4286 E. NOTA

13. A box contains 5 white balls, 3 red balls, and 7 green balls. Balls are selected at random one by one without replacement till a green ball is selected. What is the probability at least 5 balls will be selected?

A: $\frac{2}{39}$ B: $\frac{8}{429}$ C: $\frac{13}{60}$ D $\frac{6}{77}$ E. NOTA

14. Four married couples are assigned seats at random in a row consisting of eight seats. What is the probability that Mrs. And Mr. Smith end up sitting next to each other?

A: $\frac{1}{28}$ B: $\frac{1}{7}$ C: $\frac{1}{4}$ D: $\frac{1}{8}$ E. NOTA

15. A company has bought 20 machines from a manufacturer. The manufacturer advises them that 8 of these machines have a flaw. They take a random sample of 5 machines. What is the probability that exactly 2 of the machines in the sample have a flaw?

A: $\frac{2}{5}$ B: $\frac{385}{969}$ C: $\frac{5}{8}$ D: $\frac{220}{741}$ E. NOTA

16. A box contains 5 white balls, 5 red balls, and 5 green balls. Ten balls are selected at random without replacement. What is the probability that all the red balls are among the 10 selected balls?

A: $\frac{12}{143}$ B: $\frac{1}{3}$ C: $\frac{11}{140}$ D: $\frac{14}{141}$ E. NOTA

17. In a population of 100 students, 20 are seniors, 25 are juniors, and 20 are freshmen. Six of the seniors, 5 of the sophomores, and 3 of the freshman take a second language. It is given that $\frac{7}{9}$ of the upperclassmen (juniors and seniors) don't take a second language. What is the probability that a student picked at random from this group is not a senior and doesn't take a second language?

A: 0.28 B: 0.22 C: 0.72 D: 0.68 E. NOTA

18. Consider the following experiment. Each trial consists of either tossing a fair coin or rolling a fair die. You start with tossing the coin. As long as it turns out tails you will keep tossing it. The moment it turns out heads you will stop tossing it and start rolling the die. You continue rolling the die until you get a four or a five, then you start tossing the coin again. You keep applying the rules described above to change from tossing the coin to rolling the die and vice versa. What is the probability that on the third trial you will toss a coin?

A: $\frac{1}{2}$ B: $\frac{5}{12}$ C: $\frac{1}{3}$ D: $\frac{1}{4}$ E. NOTA

19. Two identical boxes are taken. The 10 letters, A up to and including J, are placed in one box, the remaining letters are placed in the other box. A box is selected at random and then a letter is taken at random from the selected box. What is the probability that the selected letter will be a vowel?

A: $\frac{17}{80}$ B: $\frac{29}{150}$ C: $\frac{15}{78}$ D: $\frac{21}{144}$ E. NOTA

20. Urn A contains two white balls and two black balls; urn B contains three white and two black balls. One ball is randomly transferred from A to B. Then one ball is drawn from B and it turns out to be white. What is the probability that the transferred ball was white.

A: $\frac{4}{7}$ B: $\frac{7}{12}$ C: $\frac{1}{2}$ D: $\frac{2}{3}$ E. NOTA

21. Four machines are in operation. Machine A produces 5 percent defective items, machine B produces 3 percent defective items, machine C produces 4 percent defective items, and machine D produces 8 percent defective items. An item is picked at random from the four machines. Given that the item picked is defective, find the probability that it came from machine B.

A: 0.03 B 0.15 C: 0.25 D: 0.10 E. NOTA

22. A student is taking a multiple-choice test consisting of 20 problems. For each problem 4 options are given of which only one is correct. To pass the test the student should have at least 11 correct answers. The student hasn't studied and selects his answers at random. What is the probability that he will pass the test?

A: 0.001 B: 0.004 C: 0.999 D: 0.715 E. NOTA

23. A die is being rolled until three sixes are obtained. What is the probability that the third six will occur at the 18^{th} roll of the die?

A: 0.167 B: 0.053 C: 0.041 D: 0.134 E. NOTA

24. Two random numbers between 0 and 5 inclusive are chosen. What is the probability that their sum is less than 3?

A: 0.2 B: 0.333 C: 0.6 D: 0.18 E. NOTA

25. In order, Anna, Beth, and Carrie take turns flipping the same fair coin. The first one to toss a head wins. What is the probability that Beth wins?

A: $\frac{1}{4}$ B: $\frac{1}{2}$ C: $\frac{1}{3}$ D: $\frac{2}{7}$ E. NOTA

26. How many people would have to be randomly selected for the probability to be greater than 50% that at least one had a birthday on February 29?

A: 254 B: 1013 C: 366 D: 730 E. NOTA

TIEBREAKER. WORK IN THE WHITE PORTION ON THE BACK OF THE SCANTRON SHEET.

The region between the parabola $y = x^2$ and the line $y = 2x$ is revolved about the line $x=2$. Find the volume swept out. What is the ratio of this volume to the probability of obtaining exactly 5 heads in 8 tosses of a fair coin. Leave answer in terms of π.

ANSWER KEY TO MU DIVISION
PROBABILITY TEST

1. A
2. D
3. C
4. C
5. A
6. A
7. E
8. B
9. B
10. C
11. C
12. C
13. A
14. C
15. B
16. A
17. D
18. B
19. A
20. A
21. B
22. B
23. C
24. D
25. D
26. B

TIEBREAKER: $\frac{256}{21}\pi$ (Answer must be left in terms of π.)