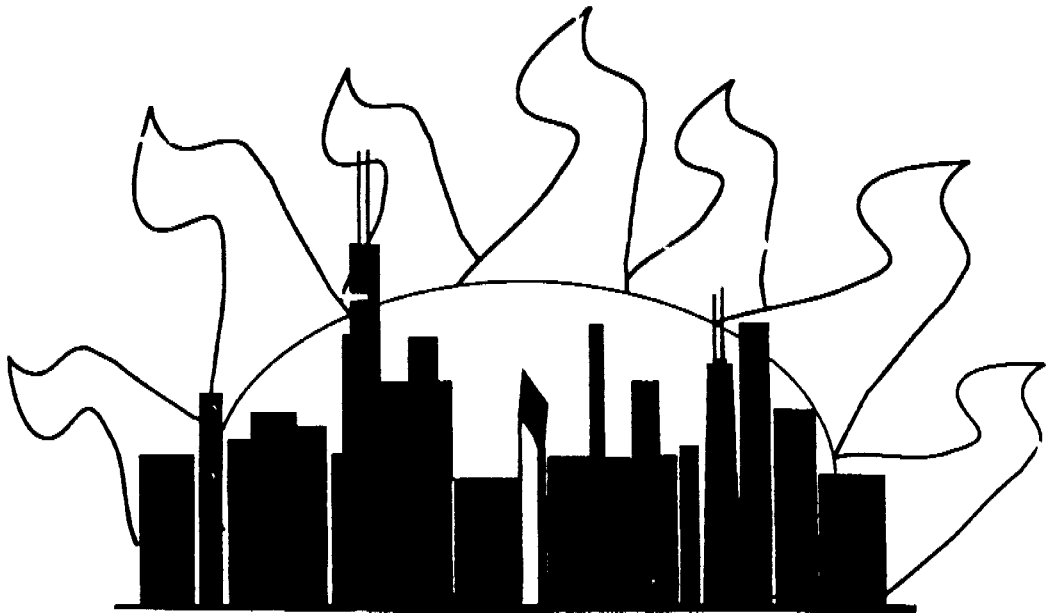# Discrete
Math

Mu Alpha Theta National Convention
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General Instructions:

Unless otherwise stated all answers should be written as decimals.

If you are asked to give your answer as a fraction, please give your answer in a/b form where a and b are relatively prime.

Questions

1.   Find the sum of x, y, z and t if $3\begin{bmatrix} x & y \\ z & t \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2t \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+t & 3 \end{bmatrix}$

2.   Let $\mathbf{u}=(2, -7, 1)$. Find $\|\mathbf{u}\|$. Give an exact answer.

3.   If $y = h(x)$, for what value of x will h^{-1} be undefined? Write your answer as a simplified fraction in lowest terms.

$$h(x) = \frac{2x-3}{5x-7}$$

4.   Suppose that 100 of the 120 math students in Ms. Azrim's class take at least one of the languages French, Italian, and Armenian. Of the students,

    65 study French   20 study French and Italian
    45 study Italian   25 study French and Armenian
    42 study Armenian   15 study Italian and Armenian

Find the number of students who study all three languages.

5.   Small for the smallest positive value of x such that $3x \equiv 5 \pmod 8$.

6.   Simplify $\dfrac{(x+1)!}{(x-1)!}$

7.   Find the number of ways that nine toys can be divided among four children so that the youngest child gets 3 toys and each of the other children receives two toys.

8.   A small town has only two food stores, A and B. Currently, each store has 50% of the total number of shoppers. The management of store A is using a new marketing plan to lure customers away from store B. The marketing director of store A expects that 90% of its customers will return next week with the other 10% going to store B and 30% of store B's customers will come to store A while 70% of store B's customers will return to store B. The management of store A would like to compute the market share for the following week, that is the percent of all shoppers who are expected to shop at store A. Give your answer as a percent.

9.   A bag contains five red marbles and six white marbles. Find the number of ways that four marbles can be drawn from the bag if two of the marbles must be red and two of the marbles must be white.

10.   A company has two machines that are used to produce a product. The first machine produces 60% of the total output and has a defective rate of 2%. The second machine produces 40% of the total output and has a defective rate of 4%. Suppose a unit of output is defective. What is the probability that it came from the first machine? Give your answer rounded to four significant digits.

11.   There are twelve points A, B, C,..., K, L in a given plane with no three points on the same line. How many triangles are determined by the points that contain the point A as a vertex?

12.   Consider four different vowels and eight different consonants. Find the number of five-letter "words" containing two different vowels and three different consonants that can be formed from the given letters.

13.   What is the value of x $A = \begin{bmatrix} -2 & 6 \\ x & -9 \end{bmatrix}$ if [A] has no inverse?

14.   There are twelve students who are eligible to attend the mathematics conference planning meeting. Find the number of ways a delegation of four students can be selected from the twelve eligible students if two of the eligible students will not attend the meeting together.

15.   If t > 0 determine t such that $\begin{vmatrix} t-2 & 3 \\ 4 & t-1 \end{vmatrix} = 0$

16. Given that $f(t) = t^2$, $g(t) = 4t$, $h(t) = t^2 - 1$, determine $f \circ (g \circ h)$ if $t = 5$.

17. Find the arithmetic mean of three consecutive positive integers whose sum is $\dfrac{1}{33}$ of

   their product.

18. Let f be the function from real numbers to real numbers defined by
$$f(x) = \begin{cases} x + 2 & \text{if 3 is a divisor of } [x] \\ x - 1 & \text{otherwise} \end{cases}$$
   $[x]$ stands for the greatest integer less than or equal to x. Compute $f(f(f(f(f(\pi)))))$.

19. The sides of a triangle are in arithmetic progression. The perimeter is 39 and the area is

   $\dfrac{39\sqrt{51}}{4}$. What is the length of the longest side of the triangle?

20. A thread manufacturer makes two types of thread, 100% cotton (C) and a cotton-
   polyester blend (P). In the manufacturing process there are two options necessary; dyeing
   time and spinning time per 100 yards of thread. C requires 2 hours of dyeing time and 2
   hours of spinning time per 100 yards of thread. P requires 3 hours of dyeing time and 10
   hours of spinning time per 100 yards of thread. The net profit for C is $10 per 100 yards;
   the net profit for P is $25 per 100 yards. If 72 hours of dyeing time and 100 hours of
   spinning time are available per week, how many yards of each type of thread should be
   manufactured to yield the largest net profit? Give your answer as the sum of the yards.