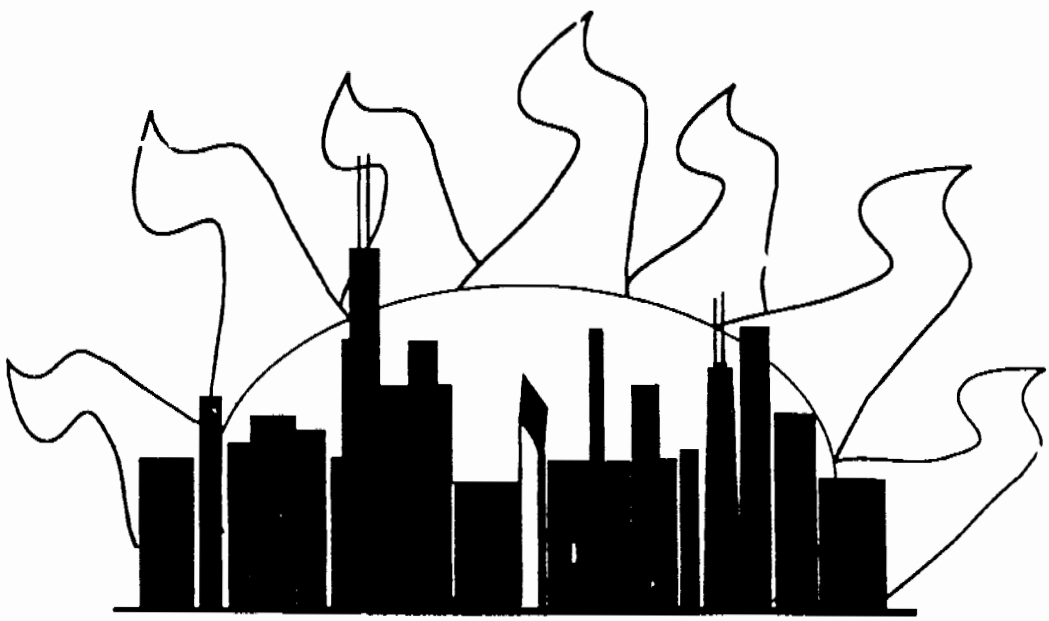
Mu Division **Topic Test 1**

Limits/ Derivatives



Mu Alpha Theta National Convention
Chicago 1998

Mu Level

General Instructions:

1. Unless otherwise stated all answers should be written as decimals.
2. If you are asked to give your answer as a fraction, please give your answer in $\frac{a}{b}$ form where a and b are relatively prime.
3. "$\frac{\partial y}{\partial x}$" indicates the partial derivative of y with respect to x.

Questions

1. Find the equation of the line tangent to the graph of $y = x^2 - 4x + 3$ when $x = 3$. For your answer write only what is to the right of the equal sign in the equation "$y = mx + b$".

2. If $x = \sum_{n=1}^{5}(2n-3)$ and $y = \sum_{n=1}^{5}(2^n - 3)$, what is the value of $x + y$?

3. Find $\lim_{u \to 0}\left(\frac{5u^2 - 4}{u+1}\right)$.

4. If $y = 2(\ln v)$ and $v = x^2$ with $x > 0$, find $\frac{dy}{dx}$.

5. Calculate the average rate of change of $y = f(x) = x^2 - 2$ between $x = 3$ and $x = 4$.

6. Find $\lim_{x \to 0}\left(\frac{x^3 - 4x}{x^2 - 2x}\right)$.

7. Find $\frac{dy}{dx}$ if $y = 4(\ln x) - 6e^x + 2x^2$. Then determine what the value of $\frac{dy}{dx}$ is when $x = 3$. Round your answer to the nearest integer.

8. Find $\lim_{x \to 2}\left(\frac{x^2 + x - 6}{x^2 + 3x - 10}\right)$. Give your answer as a simplified fraction in lowest terms.

9. If $f(x) = e^x \cos x$, evaluate $f'(1.2)$. Give answer rounded to four significant digits. Note: Use radian measure.

10. If $y = 3u^5$, find $\frac{d^2y}{du^2}$.

11. Find $\frac{\partial u}{\partial x}$ (the partial derivative of u with respect to x) when $u = x^2 + xy + y^2$.

12. Find $\lim_{x\to 3}\left(\frac{x^2+4x-21}{x^3+3x^2-16x-6}\right)$. Give your answer as a simplified fraction in lowest terms.

13. A square sheet of copper which is 20 inches on a side is to be made into an open-topped box of maximum volume. Calculate the length, in inches, of the side of one of the squares which should be cut out of each of the corners. Give your answer as a simplified fraction in lowest terms.

14. Calculate the limit: $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)$.

15. Find the exact value of $\sum_{n=0}^{5} \sin\left(\frac{n\pi}{6}\right)$.

16. Find $\lim_{n\to\infty}\left(\frac{3n^2+2n-4}{6n^2+1}\right)$.

17. Find $\frac{\partial y}{\partial z}$ when $xy = 5z$ with $x \neq 0$.

18. Find the value of $\sum_{n=1}^{999,999} \log_{10}\left(\frac{n+1}{n}\right)$.

19. To what real number does the sequence $\sqrt{6}$, $\sqrt{6-\sqrt{6}}$, $\sqrt{6-\sqrt{6-\sqrt{6}}}$ ... converge?

20. One urn contains 1 liter of water, while a second urn is empty. After $\frac{1}{2}$ of the water in the first urn is emptied into the second urn, $\frac{1}{3}$ of the water in the second urn is returned to the first urn. Then $\frac{1}{4}$ of the contents of the first urn is poured into the second urn, followed by a return of $\frac{1}{5}$ of the contents of the second urn. At each successive pouring, from alternate urns, the denominator of the fractional part poured increases by 1. How many liters of water remain in the first urn before the 1998^{th} pouring? Express your answer as a simplified fraction in lowest terms.