Advanced Word Problems

Mu Alpha Theta National Convention
Chicago 1998

General Instructions:

Unless otherwise stated all answers should be written as decimals.

If you are asked to give your answer as a fraction, please give your answer in a/b form where a and b are relatively prime.

Questions

1. On January 1, 1998 Brigit put $5000 into an account that pays 8% interest compounded quarterly, and Helen put $6000 into an account that pays 6% interest also compounded quarterly. In what year will Brigit first have more money in her account than Helen has in hers?

2. Boat A leaves point P going due north at a rate of 25 miles per hour. At the same time, boat B leaves point P going 35° to the east of north at a rate of 20 miles per hour. How long, in minutes, will it be before the two boats are 15 miles apart?

3. The half-life of a radioactive substance is 5,413 years. Find, in terms of half-lives, the time needed for a sample of the substance to decay to one-hundredth its initial size.

4. A recent survey of 100 students showed that
    8 students like asparagus
    13 students like cauliflower
    50 students like corn
    10 students like both cauliflower and corn
    2 students like asparagus and cauliflower
    1 student likes all three vegetables
    3 students like asparagus but not corn or cauliflower
    A student who likes at least one of the vegetables is chosen at random. What is the probability that she likes only corn? Give your answer as a reduced fraction.

5. Peter and Robbie are swimming laps back and forth across the length of a pool with constant speeds and essentially instantaneous turns at the ends of the pool. They start from opposite ends of the pool at the same time and meet for the first time 21 meters from one end. They continue on their ways to the ends of the pool, turn around, and meet again 8 meters from the opposite end. How long, in meters, is the pool?

6. You are given a regular five-pointed star. Find the ratio of the sum of the areas of the five shaded triangular regions to the area of the pentagonal region. Express your answer as an exact value.
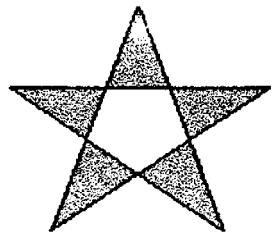
7.   The hyperbolic cosine of x, denoted cosh x, is defined by $\cosh x = \dfrac{e^x + e^{-x}}{2}$. Find the

product of all values of x such that $\cosh x = 4$.

8.   A pendulum is swung 20 cm and allowed to swing freely until it eventually comes to rest.
Each subsequent swing of the bob of the pendulum is 90% as far as the preceding swing.
How far will the bob travel before coming to rest?

9.   Find the smallest prime p such that the sum of all the reciprocals of primes up to and
including p exceeds 1.70.

10.   Find the sum of all x such that $(\log x)^2 = \ln x$.

11.   A circle has equation $x^2 + y^2 = 100$. A second circle, which is on or inside the first

circle goes through the center of the first circle and the point $(10, 0)$. The third circle,
which is in or on the second circle goes through the center of the circle and the point
$(10, 0)$. Continuing this pattern, what is the sum of all the areas of these circles? Give
your answer to the nearest tenth.

12.   The product of the ages of three teenagers is 4590. How many years old is the oldest?

13.   Jim bought two pizzas of equal thickness. One had a circumference of 50.265 and the
other a circumference of 75.398. If a 30° wedge is cut from the smaller pizza what size
wedge, to the nearest degree, should be cut from the larger so that the food volumes are
equal?

14.   Find the units digit in the following sum of binomial coefficients. $\dbinom{70}{60} + \dbinom{70}{10}$

15.   In how many ways can 3 freshmen, 4 sophomores, 2 juniors, and 3 seniors sit around a
circular table if those of the same year in school must sit together?

16.   Suppose someone tells you she has traced her family tree back 10 generations. What is
the minimum number of people on her family tree if there were no intermarriages?

17.   Two cars are on perpendicular roads. One is 30 km from the intersection. The other is 40 km from the intersection. The first car is traveling toward the intersection at 60 km/hr. The other is going away from the intersection at 50 km/hr. How many kilometers apart are they after 12 minutes?

18. V varies directly as x^2 and w^3 and inversely as y. V' is arrived at by doubling x, halving w, and tripling y. Find the ratio $\dfrac{V'}{V}$. Give your answer as a reduced fraction.

19.   You select balls from a basket. Black balls are worth 20 points, white balls are worth 12 points, and red balls are worth 3 points. What is the least number of balls you need to select to have *exactly* 97 points.

20.   Diophantus wrote the following puzzle about himself in *Anthologia Palatina.*
          Here lies Diophantus. The wonder behold--
          Through art algebraic, the stone tells how old:
          "God gave him his boyhood one-sixth of this life,
          One-twelfth more as youth while whiskers grew rife;
          And then yet one-seventh their marriage begun;
          In five years there came a bouncing new son.
          Alas, the dear child of master and sage
          Met fate at just half his dad's final age.
          Four years yet his studies gave solace from grief;
          Then leaving scenes earthly he, too, found relief"
What was Diophantus' final age?