Assume all sequences and series start at term 1, unless otherwise noted.

1. If sequence a_n is increasing and the rate at which sequence a_n is increasing is increasing, order the following from least to greatest (given that all 3 are distinct): the arithmetic mean (AM) of a_1 and a_3, the geometric mean (GM) of a_1 and a_3, and the harmonic mean (HM) of a_1 and a_3.

   A) AM, GM, HM B) AM, HM, GM C) GM, HM, AM D) HM, GM, AM E) NOTA

2. How many of the first 100 rows of Pascal's Triangle have an odd number of odd numbers?

   A) 0 B) 1 C) 50 D) 100 E) NOTA

3. $\lim_{x\to 0}\dfrac{x\cos x - e^x \sin x}{x^2}$

   A) -1 B) 0 C) $1/2$ D) 1 E) NOTA

4. Find $\dfrac{d^5}{dx^5}\tan^{-1}(x)$ at $x=0$.

   A) 5 B) -5 C) 120 D) -120 E) NOTA

5. For what values of x does the sequence $a_n = \sin^n(x)$ diverge?

   A) $x=0$ B) $x=k\pi, k\in Z$ C) $x=2k\pi, k\in Z$ D) $x=\dfrac{(2k+1)}{2}\pi, k\in Z$ E) NOTA

6. Let the polynomial function $f(x)$ contain the points $(a, \sum_{n=1}^{a}(-1)^n n^2)$, for all positive even integers a. Find $f''(2)$.

   A) $1/4$ B) $1/2$ C) 1 D) 2 E) NOTA

7. Given $\sum_{n=1}^{\infty}\dfrac{1}{n^2} = \pi^2/6$, $\sum_{n=1}^{\infty}\dfrac{(-1)^{n+1}}{n^2} =$

   A) $\pi^2/24$ B) $\pi^2/12$ C) $\pi^2/8$ D) $\pi^2/6$ E) NOTA

8. Which of the following tests cannot be used to show the convergence of $\sum_{x=1}^{\infty}\dfrac{2}{(x^2+1)}$?

   A) Integral B) Direct Comparison C) Limit Comparison
   D) P-series E) NOTA

9. $S(x)=\sum_{n=1}^{x} f(n)$ is well defined for any natural number n. $\lim_{x\to\infty} S(x)=3\pi$; find $\lim_{x\to\infty} f(x)$

A) 3π B) π C) Cannot be uniquely determined
D) Does not exist E) NOTA

10. $\sum_{x=-4}^{4} |n|+4=$

A) 4 B) 24 C) 36 D) 56 E) NOTA

11. $S_n=\sqrt{n}^{\sqrt{n}}$ for integers n. The product $(S_{-1})(S_1)$ lies on which of the following intervals:

A) $(-\infty,1)$ B) $[-1,0)$ C) $[0,1)$ D) $[1,\infty)$ E) NOTA

12. The arithmetic mean of Ψ and Ω is x^2+1 and the geometric mean is $2x+1$. If the harmonic mean of Ψ and Ω is 4, find the value of x.

A) -¼ B) ¼ C) ¾ D) Cannot be determined E) NOTA

13. Let $A=3^2+5^2+\ldots+103^2$. Let $B=2^2+4^2+\ldots+104^2$. Find $B-A$.

A) 5,457 B) 5,458 C) 5,459 D) 5,461 E) NOTA

14. Find the sum of the terms in the first 10 rows of Pascal's Triangle?

A) 1023 B) 1024 C) 2047 D) 2048 E) NOTA

15. Which of the following converges the slowest as $n\to\infty$.

A) $\sum_{n=1}^{\infty}\frac{1}{n^3}$ B) $\sum_{n=1}^{\infty}\frac{1}{n}$ C) $\sum_{n=1}^{\infty}(-e)^{-n}$ D) $\sum_{n=1}^{\infty}\frac{2n}{n^3+1}$ E) NOTA

16. $\sum_{n=0}^{\infty}\frac{2}{n^2+4n+3}=$

A) $\frac{2}{3}$ B) $\frac{5}{6}$ C) $\frac{11}{12}$ D) $\frac{3}{2}$ E) NOTA

17. a_n is an infinite geometric sequence with first term $2\cot x$ and common ratio $\sin^2 x$. b_n is an infinite geometric sequence with first term $\sin(2x)$ and common ratio $\sin^2 x$. Given that $0<x<\frac{\pi}{4}$, $\sum_{i=1}^{\infty}a_i-\sum_{j=1}^{\infty}b_j$ is on which of the following intervals?

A) $(-\infty,-1)$ B) $(-1,0)$ C) $(0,1)$ D) $(1,\infty)$ E) NOTA

For the purposes of #18-20, sequence f_n ***is defined such that*** $f_1 = 1$, $f_2 = 1$, ***and*** $f_{n+2} = f_n + f_{n+1}$.

18. As $n \to \infty$, f_n best resembles which of the following types of sequences?

A) Arithmetic B) Geometric C) Harmonic D) Constant E) NOTA

19. How many elements of the set $\{f_1, f_2, f_3 \ldots f_{100}\}$ are odd?

A) 34 B) 50 C) 66 D) 67 E) NOTA

20. The largest value of k such that $f_{k+1} \geq 2 f_k$ lies on which of the following intervals?

A) [1,9] B) [10,40] C) [41,75] D) $(75, \infty)$ E) NOTA

21. Which of the following is closest to $\prod_{n=1}^{20} \frac{n^2 - 7n - 44}{2n^3 + 4n^2 + 2n}$

A) $1/2$ B) $1/2^{20}$ C) $1/40$ D) 0 E) NOTA

22. Let $g(x)$ be the MacLauren series expansion of $\ln(3x+3)$. Evaluate $\frac{d^2}{dx^2} \int g(x)dx$ evaluated at $x = 2$.

A) 1 B) 3 C) $1/3$ D) $1/9$ E) NOTA

23. Which of the following infinite series has the greatest sum?

A) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ B) $\sum_{n=1}^{\infty} (\frac{1}{2})^n$ C) $\sum_{n=0}^{\infty} (-\frac{2}{3})^n$ D) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$ E) NOTA

24. Find the sum of an infinite geometric series with first term 42 and common ratio -2.

A) 84 B) 63 C) 56 D) 21 E) NOTA

25. Which of the following is/are true?

i. According to the Integral Test, if $\sum_{n=1}^{\infty} g(n)$ converges, it equals $\int_{x=1}^{\infty} g(x)dx$

ii. $|\sum_{n=7}^{\infty} \frac{(-1)^n}{n}| < \frac{1}{7}$

iii. $|\sum_{n=1}^{\infty} f(n)| \leq \sum_{n=1}^{\infty} |f(n)|$ if $f(x)$ is defined on reals.

iv. There exists a function $g(x)$ such that $\sum_{n=1}^{\infty} g(n)$ and $\sum_{n=1}^{\infty} \frac{1}{g(n)}$ both converge, given $g(x)$ is defined and nonzero on reals.

A) II, IV only B) II, III only C) I, II, III only D) I, II, III, IV E) NOTA

26. $\lim_{z\to\infty} \sum_{y=0}^{z} \int_{y}^{y+1} \frac{1}{x^2} dx$

A) $(\ln 2)^2$ B) $\pi^2/12$ C) $\ln 4$ D) $\pi^2/6$ E) NOTA

27. $a_1 = 1$. $a_{n+1} = 2a_n + 1$. which of the following is expressions denotes a_n?

A) $2^n + 1$ B) 2^{n-1} C) $2^n - 1$ D) 2^{n+1} E) NOTA

28. a_n is the sequence 3, 7, 13, 21, 31, 43, 57…. Let polynomial function $f(n) = a_n$, for all natural numbers n. Find $f(-\frac{1}{2})$

A) $-7/4$ B) $3/4$ C) 1 D) $7/4$ E) NOTA

29. Which of the following type of sequence is not necessarily monotonic?

A) Arithmetic B) Geometric C) Harmonic D) Constant E) NOTA

30. Find $\sum_{n=1}^{\infty} \frac{4}{2^n}$.

A) 16 B) 8 C) 6 D) 4 E) NOTA