Mu Ciphering Solutions
Number 1

The curve $y = x^2 - 1$ is revolved about the y axis to form a container. If liquid flows into the container at a rate of 2 cu. units/min., how fast is the depth of the liquid changing (in terms of pi) when the depth of the liquid is 5 units?

Answer: $\frac{2}{5\Pi}$

Solution:

$$\Delta V = \pi x^2 dy =$$

$$V = \pi \int_{-1}^{h} (y+1) dy$$

$$\frac{dV}{dt} = \frac{dV}{dh} \bullet \frac{dh}{dt} = \pi(h+1) \bullet \frac{dh}{dt}$$

$$2 = \pi(5)\frac{dh}{dt} \qquad \frac{dh}{dt} = \frac{2}{5\pi}$$

Mu Ciphering 2003
Number 2
The total area of the surface of a cylinder (including ends) is 150π ft^2. The volume is a maximum when the radius is what length.

Answer: 5 feet

Solution:

$$S.A. = 2\pi rh + 2\pi r^2$$

$$150\pi = 2\pi rh + 2\pi r^2$$

$$h = \frac{75 - r^2}{r}, \quad V = \pi r^2 h = 75\pi r - \pi r^3$$

$$\frac{dV}{dr} = 75\pi - 3\pi r^2 = 0, \quad r^2 = 25, \quad r = 5$$

Mu Ciphering 2003 Number 3

A farmer estimates that if he digs his potatoes now, he will have 160 bushels worth $2/bushel. If he waits, the crop will increase by 40 bushels/week, but the selling price will drop 20 cents per bushel, per week and picking costs will go up 5 cents per bushel, per week. When should he dig his potatoes to get the most income.

Answer: 2 weeks

Solution:

$$A = (160 + 40x)(2 - .2x - .05x) =$$

$$(160+40x)(2 - .25x)$$

$$\frac{dA}{dx} = 40 - 20x = 0, \quad x=2 \text{ weeks}$$

Mu Ciphering 2003 Number 4

A solid has a base in the form of an ellipse with major axis 10 and minor axis 8. Find the exact volume if every section perpendicular to the major axis is an isosceles triangle with altitude 6.

Answer: 60π cu. Units.

$$\frac{x^2}{25} + \frac{y^2}{16} = 1, \quad y = \frac{4}{5}\sqrt{25 - x^2}$$

Solution: The ellipse could have the equation $V = 2\int_0^5 \frac{1}{2} \bullet 6 \bullet 2y dx = 12\int_0^5 \frac{4}{5}\sqrt{25 - x^2}\, dx =$

$$\frac{48}{5}\left(\int_0^5 \sqrt{25 - x^2}\, dx\right) = \frac{48}{5}\left(\frac{1}{4}25\pi\right) = 60\pi$$

Mu Ciphering 2003 Number 5

Find the exact volume generated by revolving the area bounded by the parabola $y^2 = 8x$ and the line x=2 about the line x=2.

Answer: $\frac{256}{15}\pi$ cubic units

Solution: V=

$$2\int_0^4 \pi(2 - x)^2 dy = 2\pi\int_0^4 \left(2 - \frac{y}{8}\right)^2 dy =$$

$$2\pi\int_0^4 \left(4 - \frac{y^2}{2} + \frac{y^4}{64}\right) dy = \frac{256}{15}\pi$$

Mu Ciphering 2003 Number 6

Find $\frac{dy}{dx}$ if $\arctan\frac{y}{x} = \ln\left(x^2 + y^2\right)$.

Answer: $\frac{dy}{dx} = \frac{2x+y}{x-2y}$

Solution:

$$\frac{x\frac{dy}{dx} - y}{\frac{x^2}{y^2}+1} = \frac{1}{x^2+y^2}\left(2x + 2y\frac{dy}{dx}\right),$$

$$\frac{dy}{dx} = \frac{2x+y}{x-2y}$$

Mu Ciphering 2003 Number 7

Find $\lim_{x\to 0^+} x^x$.

Answer: 1

Solution:

$$\ln y = \lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} = \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-1}{x^2}} = 0$$

$$y = 1$$

Mu Ciphering 2003 Number 8

If the region bounded by y=x(1-x) and the x axis is rotated about the line x=2, what is the resulting volume.

Answer: $\frac{\pi}{2}$

Solution: $V = \int_0^1 2\pi(2-x)x(1-x)\,dx =$

$$\int_0^1 \left(2x - 3x^2 + x^3\right)dx = \frac{\pi}{2}$$

Mu Ciphering 2003 Number 9

Find x if $\int_1^x \frac{dt}{1+t^2} = \frac{\pi}{12}$.

Answer: $\sqrt{3}$

Solution:

$\left[\arctan t\right]_1^x = \frac{\pi}{12}$, arctan x-arctan 1=$\frac{\pi}{12}$,

arctan x-$\frac{\pi}{4} = \frac{\pi}{12}$, arctan x=$\frac{\pi}{3}$, x=$\sqrt{3}$

Mu Ciphering 2003 Number 10

In a recent survey of 400 women, one half were found to be married, one-half owned fur coats, and one-half had had their appendices removed; in addition, there were 104 married with fur coats, 116 married women with no appendices, 93 women with fur coats and no appendices, and 53 single, unfurred, appendix owning women. Find the number of de-appendicized, married women with fur coats.

Answer: 60

Solution:

Let $n(A)$ represent the numberof elements in set A.

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B)$$
$$-n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$
$$347 = 200 + 200 + 200 - 116 - 104 - 93 + x$$
$$x = 60$$

Mu Ciphering 2003 Number Practice

Evaluate the infinite product

$2^{\frac{1}{3}} \cdot 4^{\frac{1}{9}} \cdot 8^{\frac{1}{27}} \cdot 16^{\frac{1}{81}} \ldots$

Answer: $2^{\frac{3}{4}}$ or $\sqrt[4]{8}$

Solution:

$2^{\frac{1}{3}} \bullet 2^{\frac{2}{9}} \bullet 2^{\frac{3}{27}} \bullet 2^{\frac{4}{81}} \ldots = 2^{\left(\frac{1}{3}+\frac{2}{9}+\frac{3}{27}+\frac{4}{81}+\frac{5}{243}+\ldots\right)} = 2^{S}$

$\frac{S}{3} = \frac{1}{9} + \frac{2}{27} + \frac{3}{81} + \frac{4}{243} + \ldots$

$\frac{S}{3} = S - \left(\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots\right) = S - \frac{\frac{1}{3}}{1 - \frac{1}{3}} = S - \frac{1}{2}$

$S = \frac{3}{4}, \quad 2^{S} = 2^{\frac{3}{4}}$ or $8^{\frac{1}{4}}$

Mu Ciphering 2003 Number 12

Find 5a+7b+9c if

$ab = 2(a+b)$

$bc = 3(b+c)$

$ac = 4(a+c)$

Answer: 264

Solution:

$\frac{1}{2} = \frac{1}{b} + \frac{1}{a}, \quad \frac{1}{3} = \frac{1}{c} + \frac{1}{b}, \quad \frac{1}{4} = \frac{1}{a} + \frac{1}{c}$

$\frac{13}{12} = 2\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right), \quad \frac{13}{24} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$

$\frac{13}{24} = \frac{1}{2} + \frac{1}{c}, \quad c=24, \quad \text{and } a = \frac{24}{5}, \quad b = \frac{24}{7}$

$5a + 7b + 9c = 264$