Note: For all questions, answer "(E) NOTA" means none of the above answers is correct.

1. The Spiral of Archimedes can be parametrized by $x = a\theta\cos\theta, y = a\theta\sin\theta$, for $\theta > 0$. If a = 6, what is the area bound by the spiral and the x-axis between the second and third times the spiral touches the x-axis?
(A) $42\pi^3$ (B) $63\pi^3$ (C) $105\pi^3$ (D) $114\pi^3$ (E) NOTA

2. Evaluate: $\int_0^{\frac{1}{\sqrt{2}}} \frac{1}{\sqrt{1-2x^2}}dx$
(A) $\frac{\pi\sqrt{2}}{8}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi\sqrt{2}}{4}$ (D) $\frac{\pi}{2}$ (E) NOTA

3. Use a fourth-order Maclaurin series for e^x to approximate e^{-2}.
(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) 1 (E) NOTA

4. Evaluate: $\lim_{n\to\infty}\prod_{k=2}^{n}\frac{k^2}{k^2-1}$
(A) 1 (B) 2 (C) 4 (D) ∞ (E) NOTA

5. Evaluate: $\int_{-\infty}^{\infty} 2^{-|x|}dx$
(A) $\frac{\ln(2)}{2}$ (B) $\frac{1}{\ln(2)}$ (C) $2\ln(2)$ (D) $\frac{2}{\ln(2)}$ (E) NOTA

6. Suppose x and y are given by the parametric equations $x = e^{\sqrt{t}}, y = 3t - \ln(t^2)$. Find the equation of the tangent line to the curve when t = 1.
(A) $y = \frac{2}{e}x + 1$ (B) $y = \frac{2}{e}x - 1$ (C) $y = \frac{e}{2}x + 1$ (D) $y = \frac{e}{2}x - 1$ (E) NOTA

7. Compute the arc length of the parametric curve $x = 3t^2 + 1, y = 2t^3 + 4$ for $0 \le t \le 1$.
(A) $\frac{\sqrt{2}}{2}$ (B) 2 (C) $3+\sqrt{2}$ (D) $4\sqrt{2}-2$ (E) NOTA

8. Evaluate: $\int_0^{\infty}(t+3)e^{-2t}dt$
(A) 1.5 (B) 1.75 (C) 2 (D) 3.5 (E) NOTA

9. Let $L(x)=\int_0^\infty e^t e^{-xt}dt$. Given that the domain of $L(x)$ is $(1,\infty)$ find the range of $L(x)$.

(A) $(-\infty,\infty)$ (B)(0, 1) (C) $(-\infty,0)$ (D)(-1,0) (E) NOTA

10. Evaluate: $\int_{-\infty}^{\infty} \frac{x^2}{9+x^6}dx$

(A) $\frac{\pi}{9}$ (B) $\frac{\pi}{6}$ (C) $\frac{2\pi}{9}$ (D) $\frac{\pi}{3}$ (E) NOTA

11. Which of the following statements is/are true? (Note: DNE stands for "does not exist.")

I. If $\int_0^\infty f(x)dx$ DNE, then for any function $g(x)$, $\int_0^\infty f(x)g(x)dx$ DNE.

II. If $\sum_{n=0}^{\infty} a_n$ converges, and $0<\sum_{n=0}^{k} b_n<\sum_{n=0}^{k} a_n$ for all k, then $\sum_{n=0}^{\infty} b_n$ converges.

III. Any bounded monotonic sequence converges.

(A) I only (B) II only (C) II, III only (D) I, II, III (E) NOTA

12. There is exactly one value of C for which the integral $\int_0^\infty \left(\frac{x}{x^2+1}-\frac{C}{3x+1}\right)dx$ converges.

Evaluate the integral for this value of C.

(A) –ln(3) (B) –ln(2) (C) ln(2) (D) ln(3) (E) NOTA

13. Find the area of the region enclosed by the polar curve $r=3\cos(\theta)$.

(A) $\frac{3\pi}{2}$ (B) $\frac{9\pi}{4}$ (C) 3π (D) $\frac{9\pi}{2}$ (E) NOTA

14. Evaluate: $\sum_{k=1}^{\infty} \ln\left(\frac{k}{k+1}\right)$

(A) -1 (B) 0 (C) ln(2) (D) 1 (E) NOTA

15. Let $f(x)$ be a continuous positive decreasing function for $x > 0$. Also, let $a_n=f(n)$. Let $P=\int_1^6 f(x)dx$, $Q=\sum_{k=1}^{5} a_k$, and let $R=\sum_{k=2}^{6} a_k$. Which of the following statements is true?

(A) $P < Q < R$ (B) $Q < P < R$ (C) $R < P < Q$ (D) $R < Q < P$ (E) NOTA

16. Which of the following series converge?

I. $\sum_{n=1}^{\infty}\frac{|\sin(2n)|}{2^n}$ II. $1+\frac{1\cdot3}{3!}+\frac{1\cdot3\cdot5}{5!}+\ldots$ III. $\sum_{n=1}^{\infty}\frac{n+2013}{n^{2.013}}$

(A) I, II only (B) I, III only (C) II, III only (D) I, II, and III (E) NOTA

17. Evaluate: $\int_1^2 \frac{3}{x^2+3x}dx$

(A) $\ln\frac{5}{4}$ (B) $\ln\frac{8}{5}$ (C) $\ln\frac{9}{4}$ (D) $\ln 10$ (E) NOTA

18. Find the *x*-coordinate of the centroid of the region bound by the curves $y = x^2$ and $x = y^2$.

(A) $\frac{1}{20}$ (B) $\frac{3}{20}$ (C) $\frac{9}{20}$ (D) $\frac{27}{70}$ (E) NOTA

19. What is the volume of the solid formed when the triangle whose vertices are the points (1, 1), (4, 4), and (7,1) is rotated about the line $y = 4$?

(A) 9π (B) 18π (C) 36π (D) 72π (E) NOTA

20. If *x* and *y* are given parametrically by $x = 2\sin(t), y = t^2+1$, compute $\frac{d^2y}{dx^2}$ for $t=\pi$.

(A) -1 (B) -½ (C) ½ (D) 1 (E) NOTA

21. Evaluate: $\int_{e^{\pi}}^{e^{2\pi}} \frac{\sin(\ln(t))}{t}dt$

(A) -2 (B) -1 (C) 1 (D) 2 (E) NOTA

22. The area of an ellipse with major axis of length *a* and minor axis of length *b* is given by $\frac{\pi ab}{4}$. Let set *S* be the set of all ellipses such that $a + b = 6$. Find the average value of the areas of all the ellipses in *S*.

(A) $\frac{3\pi}{2}$ (B) $\frac{9\pi}{4}$ (C) 3π (D) 6π (E) NOTA

23. Evaluate: $\lim_{n\to\infty}\sum_{k=1}^{n}\sqrt[3]{\frac{k^4}{n^7}}$

(A) $\frac{1}{8}$ (B) $\frac{3}{7}$ (C) $\frac{4}{9}$ (D) $\frac{3}{4}$ (E) NOTA

24. Use the first three nonzero terms of the Maclaurin series for $\cos(x)$ to estimate the value of tan(2), using the equality $\tan^2(x)+1=\sec^2(x)$.

(A) $-2\sqrt{2}$ (B) $\frac{-2\sqrt{2}}{3}$ (C) $\frac{2\sqrt{2}}{3}$ (D) $2\sqrt{2}$ (E) NOTA

25. Find the radius of convergence of the series $\sum_{n=1}^{\infty}\frac{2^n(x-3)^n}{\sqrt{n^2+3}}$.

(A) 2 (B) 1 (C) ½ (D) 0 (E) NOTA

26. If $\int f(x)\sin(x)dx=-f(x)\cos(x)+\int 3x^2\cos(x)dx$, which of the following could $f(x)$ be?

(A) $3x^2$ (B) x^3 (C) $-x^3$ (D) $\cos(x)$ (E) NOTA

27. Which of the following rules of differentiation is most closely related to the Integration by Parts Formula? (Note: For those of you who want to dispute this because "most" isn't a very precise term…think twice!)

(A)Chain Rule (B) Sum Rule (C) Quotient Rule (D) Product Rule (E) NOTA

28. Let L be the arc length of the curve $f(x)=\ln(\cos(x))$ on the interval $0<x<\frac{\pi}{4}$. Find e^L.

(A) $\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{2}+1}{2}$ (C) $\frac{\sqrt{2}+2}{2}$ (D) $\sqrt{2}+1$ (E) NOTA

Use the following information for questions 29-30:
One of the most useful applications of BC Calculus is to Probability Theory. In particular, one is often interested in computing properties such as the Expected Value (i.e., the mean) of a distribution with some *Probability Density Function* (PDF) $f(x)$. A function $f(x)$ is a PDF if $\int_{-\infty}^{\infty} f(x)dx = 1$ and $f(x) \geq 0$ for all x.

29. The **Expected Value** (denoted $\mathsf{E}(X)$) of a distribution with PDF $f(x)$ is $\int_{-\infty}^{\infty} xf(x)dx$. Find the expected value of the distribution with PDF $f(x) = \begin{cases} ce^{-cx}, x \geq 0 \\ 0, x < 0 \end{cases}$.

(A) $\frac{1}{c^2}$ (B) $\frac{1}{c}$ (C) 1 (D) c (E) NOTA

30. The **Variance** of a distribution with PDF $f(x)$ is defined to be $\mathsf{E}\left(X^2\right) - \left(E(X)\right)^2$. Given that for any continuous function g we have $\mathsf{E}(g(X)) = \int_{-\infty}^{\infty} g(x)f(x)dx$ find the variance of the distribution with PDF $f(x) = \begin{cases} ce^{-cx}, x \geq 0 \\ 0, x < 0 \end{cases}$.

(A) c (B) c^2 (C) $\frac{2}{c^2}$ (D) $\frac{1}{c^2}$ (E) NOTA