Mu Alpha Theta National Convention: Denver, 2001
Number Theory Topic Test – Mu Division

1. For how many ordered pairs of integers (m,n) does m multiplied by n equal 840?

   (A) 16 (B) 48 (C) 64 (D) 96 (E) NOTA

2. Find the sum of the positive integral proper factors of 15625.

   (A) 625 (B) 3,281 (C) 3,781 (D) 3,906 (E) NOTA

3. In how many consecutive zeros does the number 15634! end?

   (A) 3,282 (B) 3,782 (C) 3,907 (D) 4,032 (E) NOTA

4. What is the smallest odd integer greater than 10,000 with exactly 32 positive integral factors?

   (A) 10,105 (B) 10,395 (C) 15,015 (D) 23,205 (E) NOTA

5. Find the largest integer d for which there are no nonnegative integer solutions (a, b, c) which satisfy the equation

   $$5a + 7b + 11c = d$$

   (A) 8 (B) 9 (C) 13 (D) 17 (E) NOTA

6. What is the least common multiple of the numbers 91, 111, and 297?

   (A) 272,727 (B) 999,999 (C) 2,999,997 (D) 333,333 (E) NOTA

7. What is the sum of the positive prime divisors of 82,861?

   (A) 121 (B) 131 (C) 2,061 (D) 1,810 (E) NOTA

8. If 432_8 is equal to 189_n, what is n?

   (A) 13 (B) 14 (C) 21 (D) 23 (E) NOTA

9. What is the smallest positive prime number that leaves a remainder of one when divided by both 3 and 11?

   (A) 23 (B) 29 (C) 43 (D) 67 (E) NOTA

10. How many of the first 400 Fibonacci numbers are multiples of 3? (Let the first two Fibonacci numbers both be 1)

(A) 50 (B) 99 (C) 100 (D) 133 (E) NOTA

11. Which of the following is congruent to 7 (mod 13)?

(A) 280 (B) 550 (C) 650 (D) 680 (E) NOTA

12. When the digits of a positive two-digit integer are reversed, the resulting number is 27 more than the original number. Find the difference when the tens digit of the original number is subtracted from the units digit of the original number.

(A) 1 (B) 3 (C) 4 (D) 9 (E) NOTA

13. Which of the following numbers is equal to 121_8?

(A) 144_7 (B) 155_6 (C) 1010101_2 (D) 88_9 (E) NOTA

14. What is the sum of the positive integral factors of 840?

(A) 1,440 (B) 2,775 (C) 2,880 (D) 3,000 (E) NOTA

15. Let $A = \sum_{q=1}^{32} (q!)$. What is the remainder when A is divided by 144?

(A) 9 (B) 10 (C) 11 (D) 12 (E) NOTA

16. If B is a positive integer and $B \equiv 2 \pmod 3$ and $B \equiv 7 \pmod 8$, what is the remainder when B is divided by 12?

(A) 1 (B) 5 (C) 8 (D) 11 (E) NOTA

17. What is the sum of all the positive perfect cubes less than 100,000?

(A) 1,168,561 (B) 1,119,364 (C) 1,071,225 (D) 1,272,384 (E) NOTA

18. What is the remainder when $(15^3)(10^5)$ is divided by 128?

(A) 32 (B) 64 (C) 96 (D) 48 (E) NOTA

19. If $40a \equiv 1 \pmod 7$, what is $162a$ congruent to (mod 7)?

(A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

20. The 5-digit number $5A55B$ is divisible by 72. What digit does A represent?

(A) 4 (B) 1 (C) 6 (D) 3 (E) NOTA

21. Katie Holmes shares the same birthday as the oldest living pinstriped fuzzy tortoise, Bart (who is between 400 and 500 years older than Katie). For seven consecutive birthdays, Bart was an integer number of times as old as Katie. How old was Bart on the seventh of those birthdays?

(A) 448 (B) 427 (C) 441 (D) 504 (E) NOTA

22. A positive integer, N, has a remainder of 1 when divided by 3, a remainder of 3 when divided by 5, a remainder of 5 when divided by 7, and a remainder of 7 when divided by 9. What is the smallest possible value of N?

(A) 943 (B) 1,258 (C) 648 (D) 313 (E) NOTA

23. What is the hundreds digit of 7^{404}?

(A) 0 (B) 1 (C) 2 (D) 4 (E) NOTA

24. How many whole numbers are there less than 10,000 which have units and tens digits of 1 when expressed in bases 4, 5, and 6?

(A) 0 (B) 1 (C) 83 (D) 42 (E) NOTA

25. If $11x \equiv 65 \pmod{67}$, which of the following could not be x?

(A) 79 (B) 3,965 (C) 6,567 (D) 6,712 (E) NOTA

26. Find the sum of the 50 smallest natural numbers greater than one which each have an even number of positive integral proper factors.

(A) 125,000 (B) 45,526 (C) 42,925 (D) 1,625,625 (E) NOTA

27. What is the second smallest positive integer, x, such that $x \equiv 2 \pmod 4$, $x \equiv 3 \pmod 9$, and $x \equiv 5 \pmod{25}$?

(A) 930 (B) 480 (C) 1,830 (D) 33,330 (E) NOTA

28. What is the second smallest positive integer that is a multiple of 4 and has no digit greater than 1 when expressed in base 5?

(A) 112 (B) 312 (C) 656 (D) 756 (E) NOTA

29. The product of two prime numbers between 60 and 80 is one less than a perfect square. What is the greater of the two primes?

(A) 67 (B) 71 (C) 79 (D) 73 (E) NOTA

30. A number, N, expressed in base $(A+1)$ is $AAAA$. If $N = Q(Q-2)$, what is Q expressed in base $(A+1)$?

(A) 101 (B) 10A (C) 111 (D) 1A1 (E) NOTA

31. For how many values of b, $0 < b < 24$, could $p^2 \equiv b \pmod{24}$ for some prime number p?

(A) 3 (B) 4 (C) 6 (D) 23 (E) NOTA

32. Let P be the product of some 8 consecutive triangular numbers. If $P \equiv N \pmod{128}$ where N is a whole number less than 128, what is the remainder when N is divided by 64?

(A) 0 (B) 16 (C) 32 (D) 48 (E) NOTA

33. A counting number, n, is "stupid" if the sum of the first n positive perfect cubes is a perfect square. A number is a "political candidate" if it is both even and composite. How many of the first 50 even counting numbers are stupid political candidates?

(A) 1 (B) 50 (C) 25 (D) 49 (E) NOTA

34. On the television station MTV, A% of the airtime is devoted to the airing of quality music, B% to the airing of other music, C% to the airing of advertisements, and D% to the airing of other programming. A, B, C, and D are positive integers and add up to 100. $A < B$, C is twice B, and D is four times C. If D is a multiple of 9, find A.

(A) 0 (B) 1 (C) 4 (D) 10 (E) NOTA

35. What is the remainder when 5^{201} is divided by 11?

(A) 1 (B) 3 (C) 4 (D) 5 (E) NOTA

36. A positive integer has 32 digits when expressed in base 2. How many digits are there in the base 10 representation of that number?

(A) 10 (B) 11 (C) 12 (D) 13 (E) NOTA

37. N is a positive integer less than one hundred. If $3^N \equiv N \pmod{100}$, what is N?

(A) 7 (B) 49 (C) 87 (D) 41 (E) NOTA

38. Let K be a positive integer such that $K \equiv 1 \pmod{3}$, $K \equiv 3 \pmod{5}$, and $K \equiv 7 \pmod{11}$. What is the sum of the 20 smallest possible values for K?

(A) 32,810 (B) 36,110 (C) 31,350 (D) 43,320 (E) NOTA

39. Let $F_{n+1} = F_n + F_{n-1}$ and let $F_1 = F_2 = 1$. Find the smallest positive integer, m, such that $F_{n+m} \equiv F_n \pmod{9}$ for all integers, n.

(A) 8 (B) 12 (C) 20 (D) 24 (E) NOTA

40. A set consists of three relatively prime positive integers, the product of which is 720. What is the smallest possible sum of these numbers?

(A) 22 (B) 27 (C) 30 (D) 62 (E) NOTA